Strongly coupled SYM plasmas and String Theory

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Workshop on phenomenology beyond the Standard Model in the LHC era

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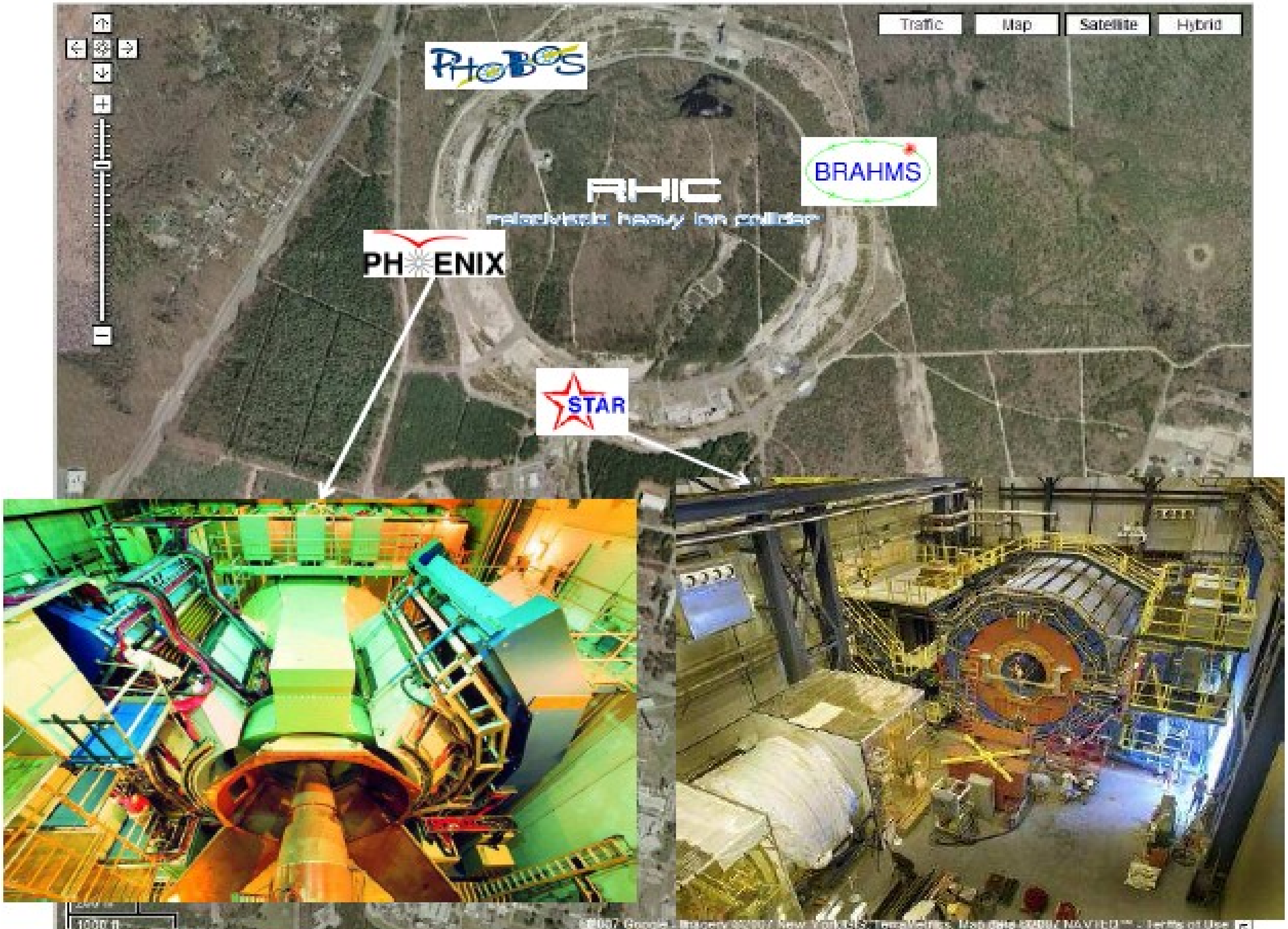
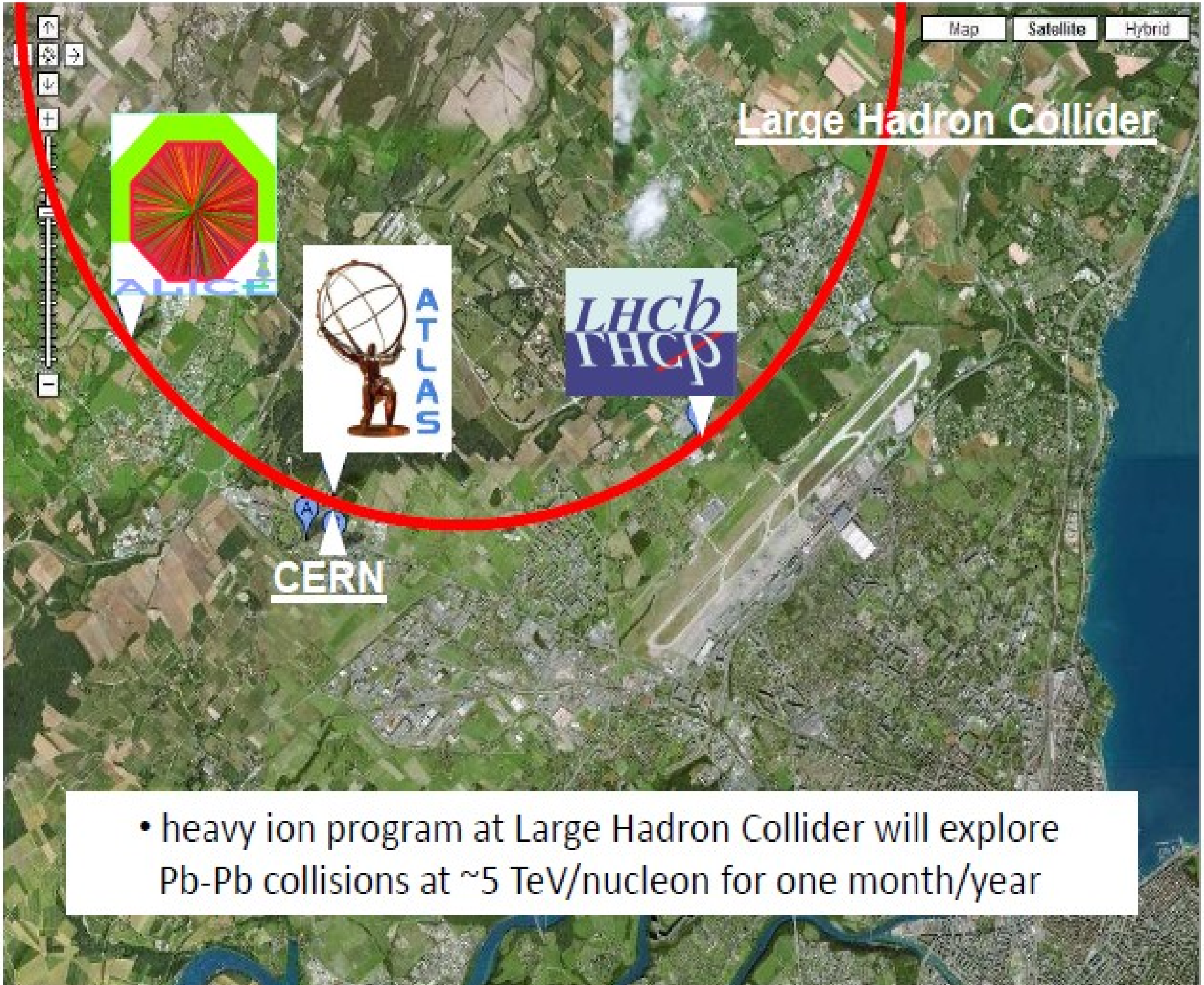
Syllabus

- Heavy ion collision phenomenology and lattice results.
- Bulk properties of strongly coupled plasma from the gauge/string duality I: introduction, thermodynamics, and mass and charge transport properties.
- Bulk properties of strongly coupled plasma from the gauge/string duality II: parton energy loss, jet quenching parameter, string theory corrections. Adding flavour. Thermalisation. Hadronisation and Jets. Outlook.

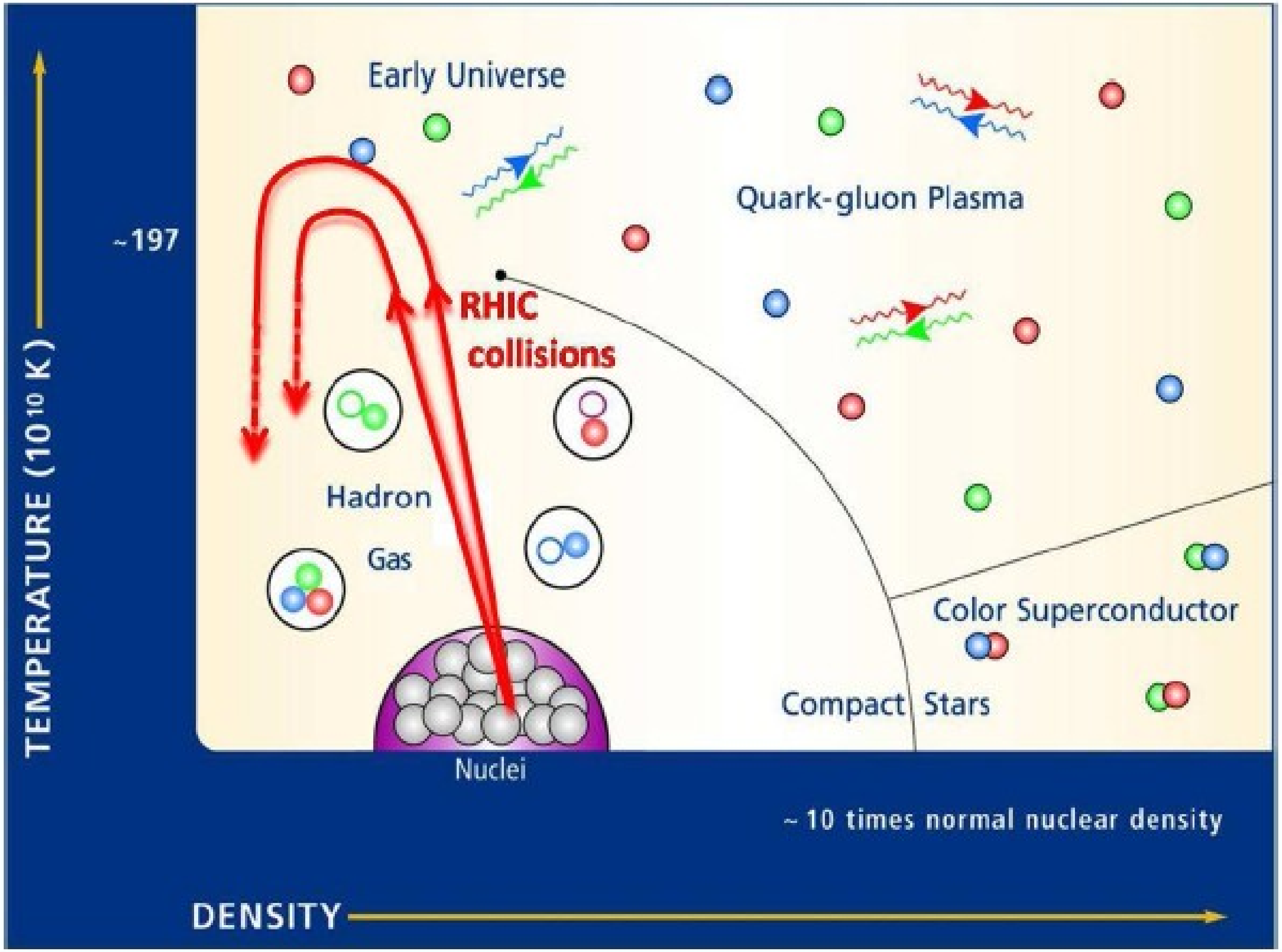
Relativistic Heavy Ion Collider: RHIC





• heavy ion program at Large Hadron Collider will explore Pb-Pb collisions at ~5 TeV/nucleon for one month/year

The QCD Phase Diagram



The RHIC (last few years) & LHC (2010-2011) results

- Deconfined quark-gluon plasma (QGP) is produced from heavy ion collisions.

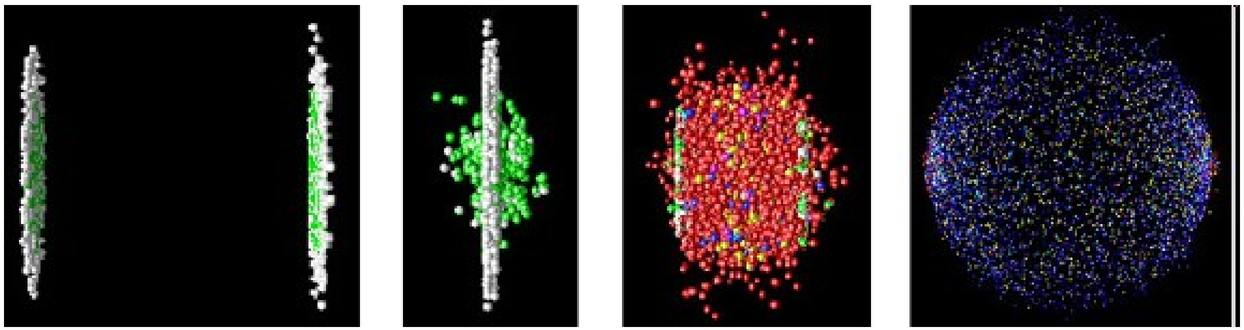
- Data suggest it corresponds to a strongly coupled regime of QCD.

- Plasma behaves like an ideal fluid.

- Excellent environment to apply the gauge/string duality at full extent: supergravity + string corrections.

A view of the Au-Au collitions at RHIC - simulation

Time arrow



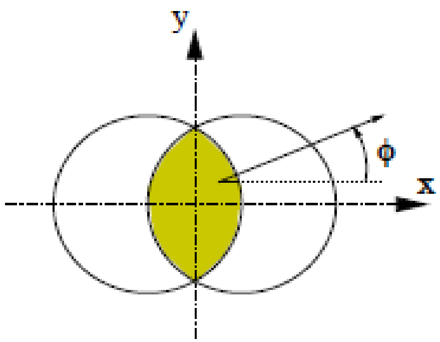
Approach: collisions of two Au nuclei, which look flattened by relativistic effects. E~100 GeV/nucleon

Thermalization: Part of the kinetic energy converted to intense heat, quarks and gluons deconfine. Timescale ~ 2 x10^{-22} seconds

Expansion: Quark-gluon plasma exhibits collective flow described by hydrodynamics (Elliptic flow)

Hadronization: Expansion and cooling, then matter converted to hadrons

How many particles are produced in a typical heavy ion collision?



Define pseudorapidity: $\eta = -\log(\tan(\theta/2))$

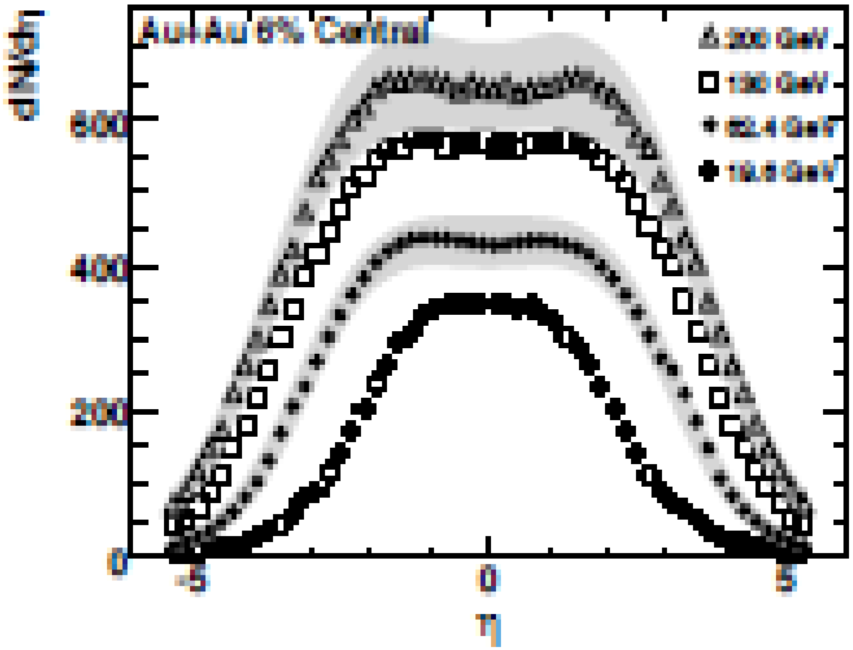
$s = p_1^2 + p_2^2$

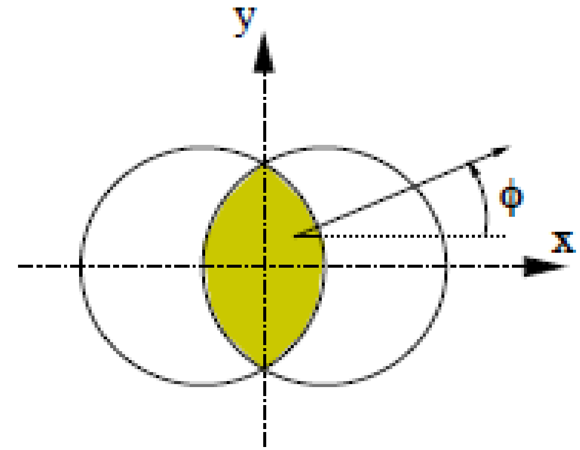
The top energy achieved at RHIC is $\sqrt{s} = 200$ GeV per nucleon at the center of mass.

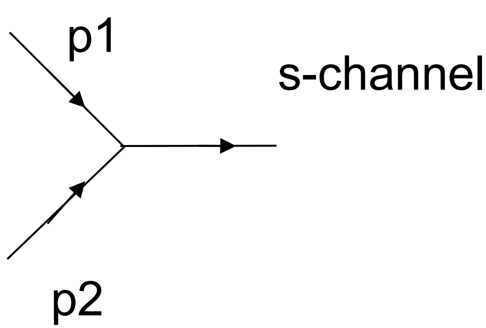
^{197}Au $\Rightarrow 197 \times 200 \overset{\text{GeV}}{\text{MeV}} \sim 40$ TeV.

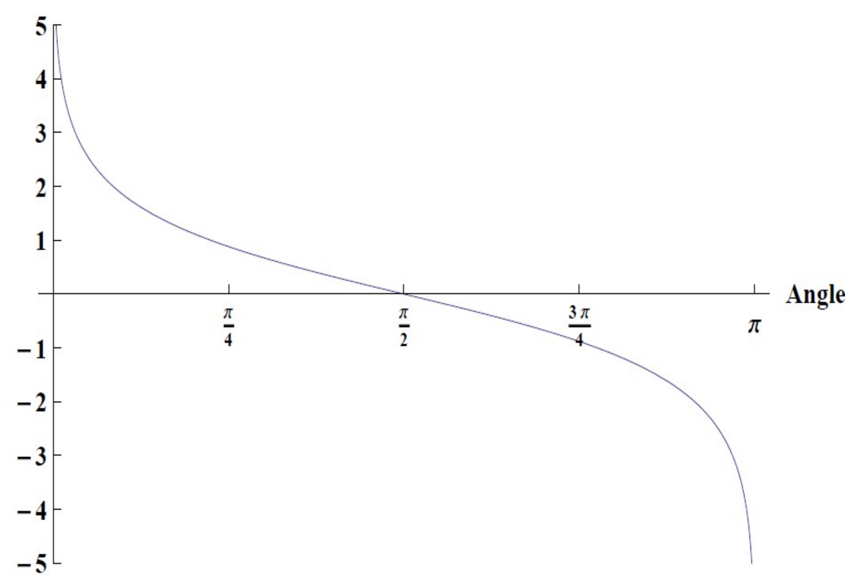
Integrating under the curve for 200 $\overset{\text{GeV}}{\text{MeV}}$ it gives 5060 ± 250 charged particles. If consider pions $\pi^{\pm}$ but also π^0 $\Rightarrow$ a factor 3/2 appears $\Rightarrow$ 8000 particles at RHIC inthe final state!

Multiplicity grows with energy and curves are centered around $\eta = 0 \Rightarrow \theta = \pi/2$.


Au+Au 6% Central
200 GeV
130 GeV
62.4 GeV
19.6 GeV
dN/dη
0
200
400
600
-5
0
5
η


y
x
φ


p1
p2
s-channel

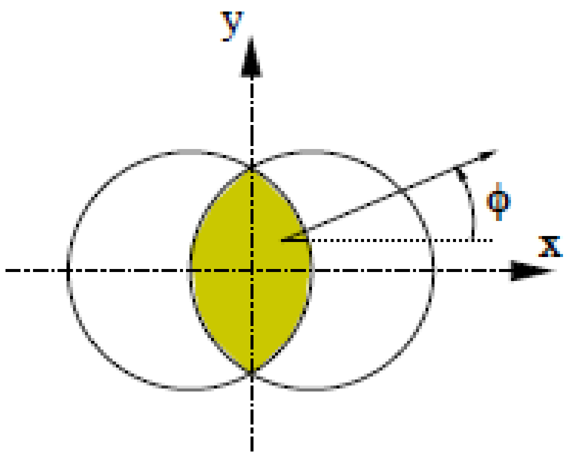

5
4
3
2
1
−1
−2
−3
−4
−5
π/4
π/2
3π/4
π
Angle

We will discuss about:

- Elliptic flow $\Rightarrow$ how soon after the collision matter moving collectively form and constrains the value of the shear viscosity.
- Jet quenching: how this matter affects and it is affected by a high-velocity coloured particle plowing through it.
- Suppression of quarkonium production: characterizes temperature and screen between coloured particles.

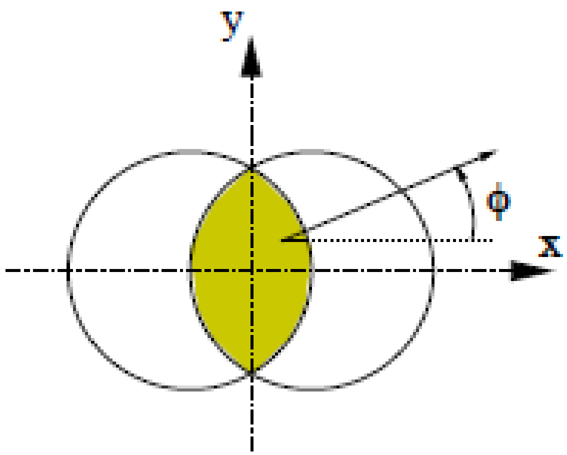
The reason to do heavy ion collisions is to create as large a volume as possible of matter at high energy density $\neq$ of $p + p$ collisions.

Elliptic flow: phenomenology



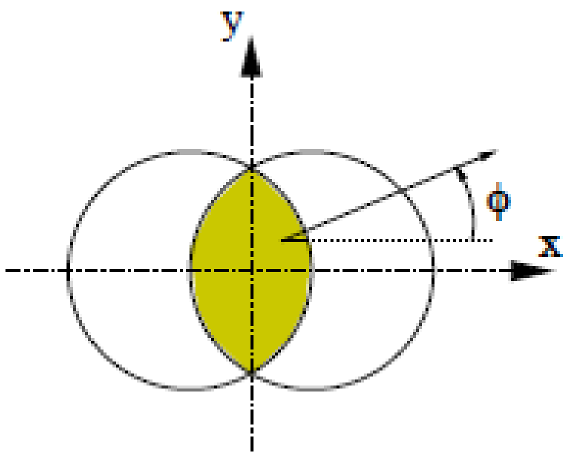
- Suppose the impact parameter comparable to the nucleus radius.
- Collision of 2 pancakes (due to relativistic Lorentz contraction).
- Almond shape region of collision.
- Nucleons outside the almond DON'T collide and are on the beam pipe.
- Now, if the few hundred particles produced by these nucleon-nucleon collisions came from independent nucleons (like p+p) by the central limit theorem they would have been uniformly distributed with ϕ.
- Otherwise, if the came from nucleons in the same nucleus $\Rightarrow$ non-uniform distribution. And data shows this!

Elliptic flow: phenomenology



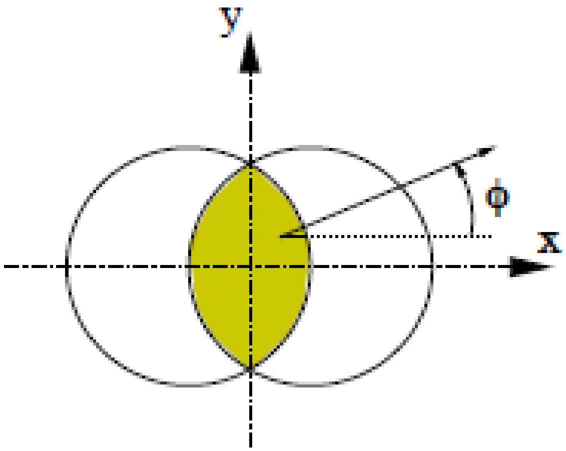
- Collisions within the almond shape re-scatter $\Rightarrow$ reach local equilibrium.
- This behaves like a kind of fluid $\Rightarrow$ it determines the shape of the azimutal distribution after the collision occurs.
- This flow is ELLIPTIC: longer direction transversal to the beam direction.
- Evolution as a drop with no external pressure and high internal pressure.
- The hydrodynamical model works well if η/s is very small.
- This is an indication about the existence of a strongly coupled fluid!

Elliptic flow: phenomenology



- 1 fm after the collision $\Rightarrow$ there is a hydrodynamical behaviuor characterized by thermodynamical equilibrium.
- The energy density is well above the one of the hadron-QGP crossover.
- This justifies the claim that heavy ion collisions produce QGP.
- Then the QGP is STRONGLY COUPLED, low η/s.
- pQFT fails to describe this system.
- Gauge/string duality does a better job. More about this later.

Elliptic flow: phenomenology



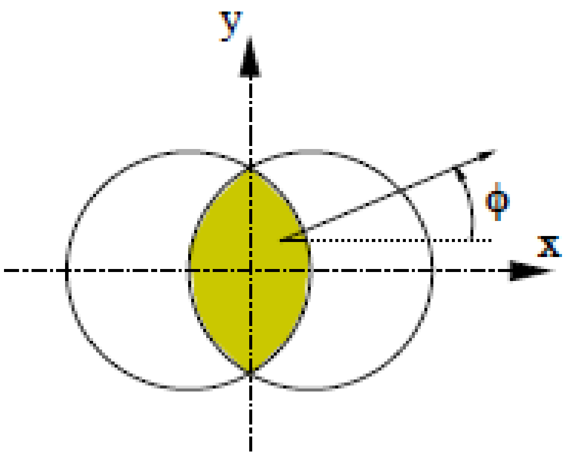
$$\mathbf{p} = \left(p_T \cos\phi, p_T \sin\phi, \sqrt{p_T^2 + m^2} \sinh y \right)$$

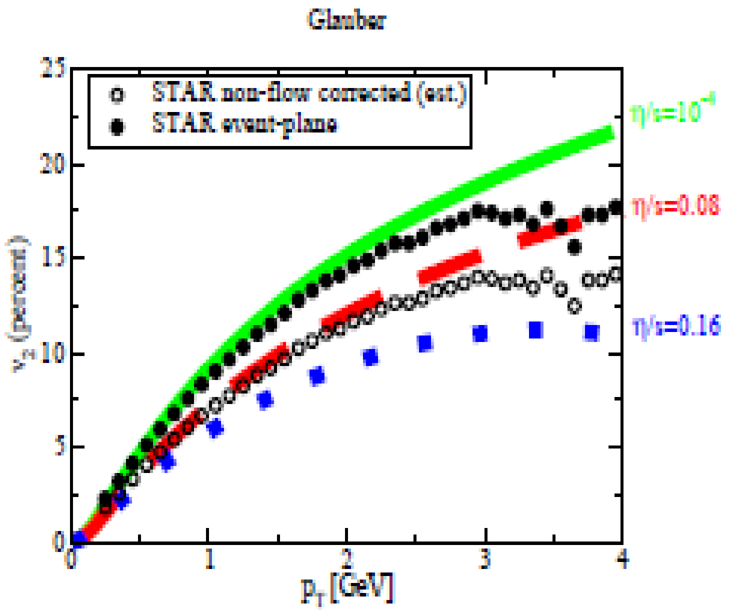
$$E = \sqrt{p_T^2 + m^2} \cosh y$$

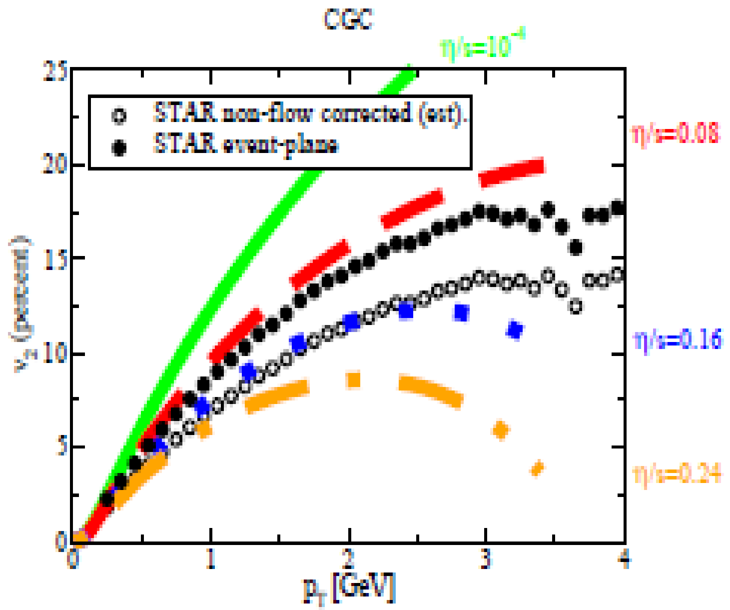
$$\frac{\mathrm{d}N}{\mathrm{d}^2\mathbf{p}_t \, \mathrm{d}y} = \frac{1}{2\pi p_T} \frac{\mathrm{d}N}{\mathrm{d}p_T \, \mathrm{d}y} \left[1 + 2v_1 \cos(\phi - \Phi_R) + 2v_2 \cos 2(\phi - \Phi_R) + \cdots \right] ,$$

$$v_n \equiv \langle \exp\left[i\, n\, (\phi - \Phi_R)\right] \rangle = \frac{\int \frac{dN}{d^3\mathbf{p}} \, e^{i\, n\, (\phi - \Phi_R)} \, d^3p}{\int \frac{dN}{d^3\mathbf{p}} \, d^3p}$$

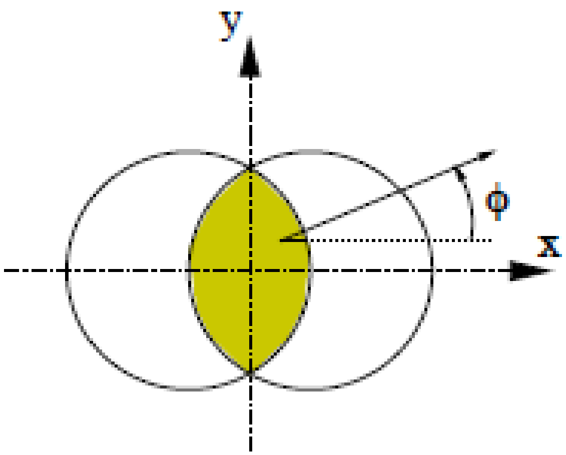
Elliptic flow: phenomenology







Elliptic flow: phenomenology



Ideal hydrodynamics: all gradient terms are neglected

$$\partial_\mu T^{\mu\nu} = 0\,, \qquad \partial_\mu J^\mu_B = 0\,,$$

$$u_\mu T^{\mu\nu} = -\varepsilon u^\nu$$

$$T^{\mu\nu} = T^{\mu\nu}_{\rm ideal} + \Pi^{\mu\nu} \qquad T^{\mu\nu}_{\rm ideal} = (\varepsilon + P) u^\mu u^\nu + P g^{\mu\nu}\,,$$

First order dissipative fluid dynamics

$$\Pi^{\mu\nu} = -\eta(\varepsilon)\sigma^{\mu\nu} - \zeta(\varepsilon)\Delta^{\mu\nu}\,\nabla\cdot u\,, \qquad \nabla^\mu = \Delta^{\mu\nu} d_\nu$$

$$\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu\,,$$

$$\sigma^{\mu\nu} = \Delta^{\mu\alpha}\Delta^{\nu\beta}\left(\nabla_\alpha u_\beta + \nabla_\beta u_\alpha\right) - \frac{2}{3}\Delta_{\alpha\beta}\nabla\cdot u$$

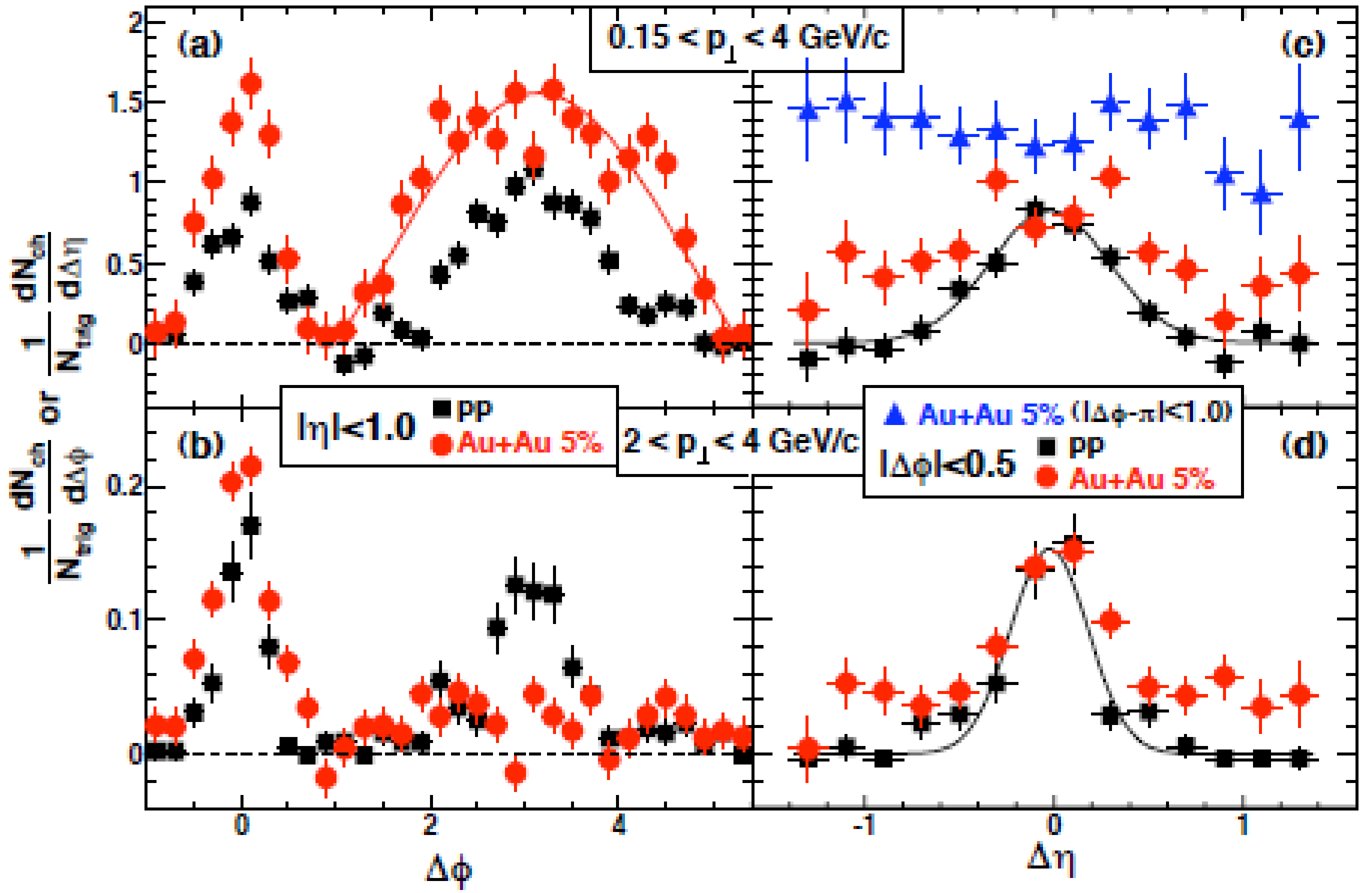
Second order dissipative fluid dynamics

$$\tau_\Pi D\Pi^{\mu\nu} = -\Pi^{\mu\nu} - \eta\sigma^{\mu\nu}\,,$$

Jet quenching: phenomenology

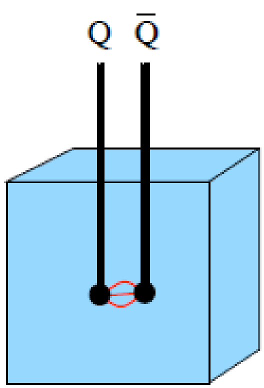
- What happens to a (hard) parton (quark or gluon) with momentum $>> T$ plough in the plasma.
- Recall this is an internal probe, produced by the collision itself (different from say DIS).
- The (hard) parton losses energy.
- Strongly coupled fluid response to this parton.
- Change in the direction of momentum: transverse momentum broadening.
- Transverse here means perpendicular to the momentum of this hard parton (not to the beam).

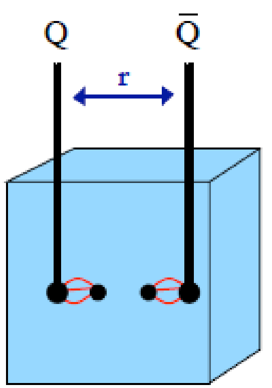
Jet quenching: phenomenology



Quarkonia in hot matter

- It is an operational way to think of the QGP deconfinement.
- What prevents the formation of a meson within QGP?
- Answer: screening due to the presence of QGP between the quark and the anti-quark.
- Then, it suggests the question: how close together do the quark and anti-quark to be in order for their attraction not to be screened?
- Similarly: how close together do the quark and anti-quark to be in order for them to feel the same attraction they would feel if they were in vacuum?
- In 1986 Matsui and Satz suggested quarkonia should be a good test as they are significantly suppressed compared with other lighter mesons or baryons.
- J/Ψ, Ψ', χ_c mesons, etc. from charmonium.
- Υ, Υ' mesons, etc. from bottomonium.
- There are lattice calculations about this.





Lattice QCD: some results

$$G_R^{xy,\,xy}(t,x) = -i\Theta(t) < [T^{xy}(t,x)T^{xy}(0,0)] >$$
$$G_R^{J,\,J}(t,x) = -i\Theta(t) < [J(t,x)J(0,0)] >$$

J is the conserved current associated to baryon number, strangeness or electric charge in QCD; of R-symmetry in SYM.

Then, via Green-Kubo relations (more later) we have:

$$\eta = -\lim_{\omega\to 0} \frac{ImG_R^{xy,\,xy}(\omega,\kappa=0)}{\omega}$$
$$\sigma = D\chi = -\lim_{\omega\to 0} \frac{ImG_R^{J,\,J}(\omega,\kappa=0)}{\omega}$$

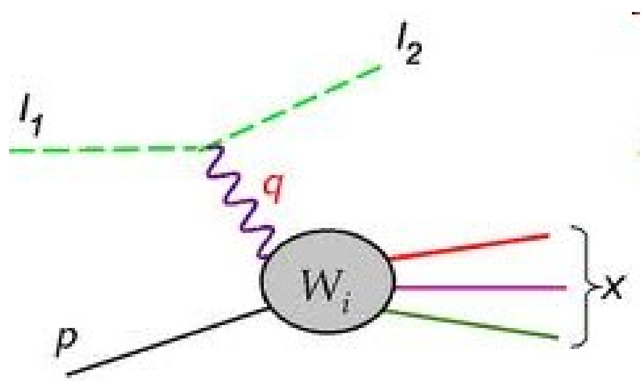
Lattice QCD: some results for η/s

- $\eta/s = 0.134(33)$ for $T = 1.65T_c$ H. Meyer 2006
- $\eta/s = 0.102(56)$ for $T = 1.24T_c$ H. Meyer 2006
- From AdS/CFT: Kovtun, Son, Starinets bound (2002) $\eta/s = 1/4\pi \simeq 0.08$
- Results on electrical conductivity to be discussed in the third lecture.

Motivation I: DIS

The DIS amplitudes for electron-hadron scattering can be extracted from the matrix element of two electromagnetic currents inside the hadron, which defines the hadronic tensor:

$$W_{\mu\nu} \equiv i \int d^4y\, e^{iq\dot{y}} < P, \mathcal{Q}|[J_\mu^{em}(y), J_\nu^{em}(0)]|P, \mathcal{Q} > \; .$$



Motivation II: DIS off strongly coupled plasmas

- Probing the plasma at momenta/energy $>> T$, with $T > T_c$, we are looking at the DIS regime.
- Take $\mathcal{N} = 4$ SYM theory at finite T. DIS corresponds to scattering off thermal excitations in the plasma.
- The holographic dual is a Schwarzschild-AdS$_5$ b.h. times S^5.
- This metric gets corrections α'^3.
- We calculated their effects on "thermal structure functions" and find enhancement.

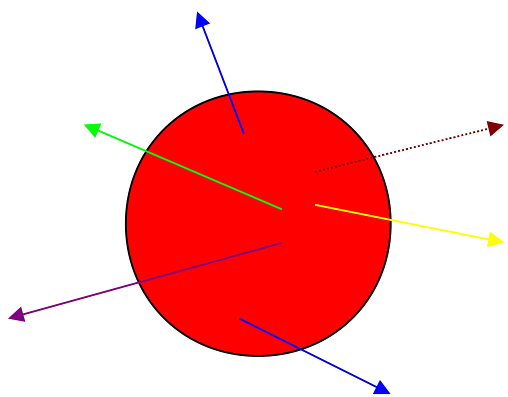
Motivation III: QGP Hydrodynamics: $E < T$

The transport coefficients.

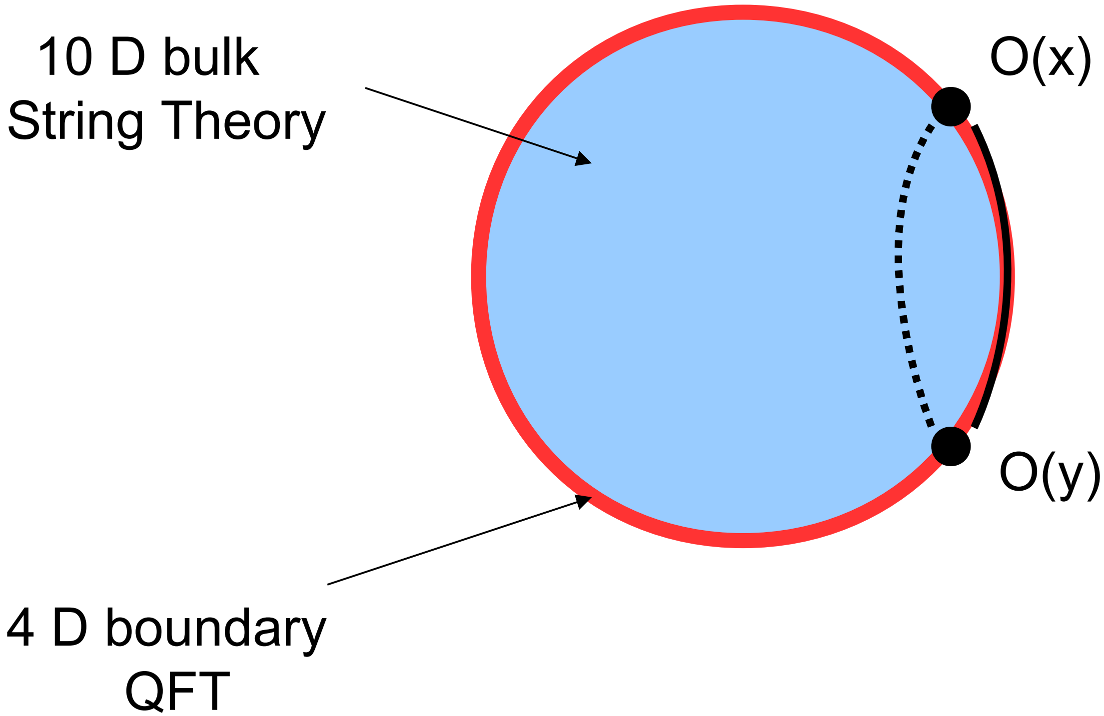
- Mass transport: shear viscosity and diffusion constant.
- Charge transport: conductivity and charge diffusion.

Motivation IV: Plasma photo-production

- Plasma in thermal equilibrium, but optically thin.
- Photons are emitted from the plasma, which does not include prompt photons produced by the initial scattering of partons from the colliding nuclei.
- The electromagnetic coupling constant, e, is consider small enough to ensure photons are not to be re-scattered and consequently do not thermalise.



AdS/CFT: The Idea



<O(x)O(y)>

The Euclidean generating functional of the connected Green's functions:

$$Z[J] =< 0|\hat{T}\{e^{\frac{-1}{\hbar}\int d^4\mathcal{L}_{int}}e^{\frac{-1}{\hbar}\int d^4x\hat{\phi}(x^{\mu})J(x^{\mu})}\}|0>$$

For n-point QFT correlators we have

$$< 0|\hat{T}\{\hat{\phi}(x_1^{\mu})\cdots\hat{\phi}(x_n^{\mu})\}|0>\equiv$$

$$\frac{(-1)^n}{Z[J]}\frac{\delta^n < \exp(-\int\hat{\phi}(x^{\mu})J(x^{\mu})) >}{\delta J(x_1^{\mu})\cdots\delta J(x_n^{\mu})}|_{J\to 0}$$

$$\frac{(-1)^n}{Z[J]}\frac{\delta^n Z[J]}{\delta J(x_1^{\mu})\cdots\delta J(x_n^{\mu})}|_{J\to 0}$$

QCD:
Nc=3=Nf,Matter in the fundamental representation, Confinement, Chiral symmetry breaking, Discrete spectrum …

N=4 SYM:
Nc large, Matter in the adjoint Representation, Deconfined Conformal, Supersymmetric…

At T=0 they are very different theories

QCD:
Strongly coupled gluons and fundamental matter,
Deconfined, screening, finite Correlation lengths.

N=4 SYM: Strongly coupled gluons and adjoint (and fundamental) matter,
Deconfined, screening, finite Correlation lenths.

At T>Tc they are very similar theories

QCD:
Runs to weak coupling, leading to a free gas of quarks and gluons.

N=4 SYM:
Coupling remains strong: strongly coupled plasma.

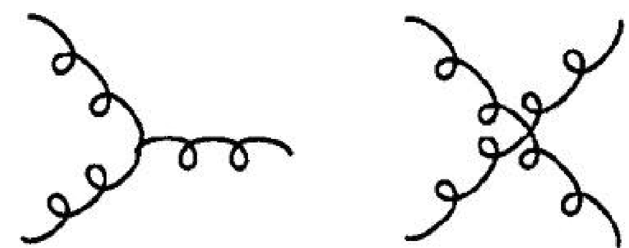
At T>>Tc they are very different theories

Yang-Mills theories: Non-Abelian gauge symmetry group $SU(N)$

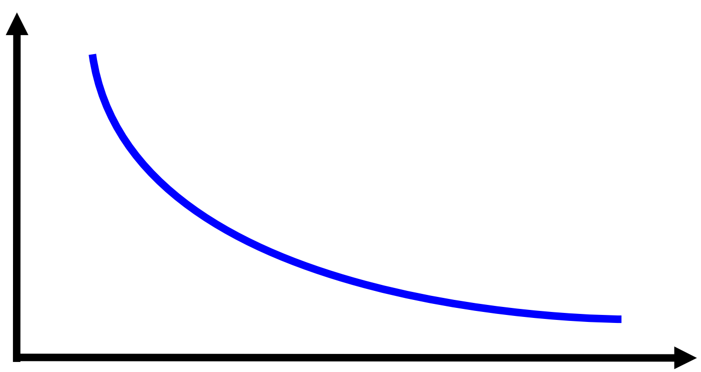
$$A_\mu \rightarrow U(x) A_\mu U^\dagger(x) + (\partial_\mu U(x)) U^\dagger(x)$$

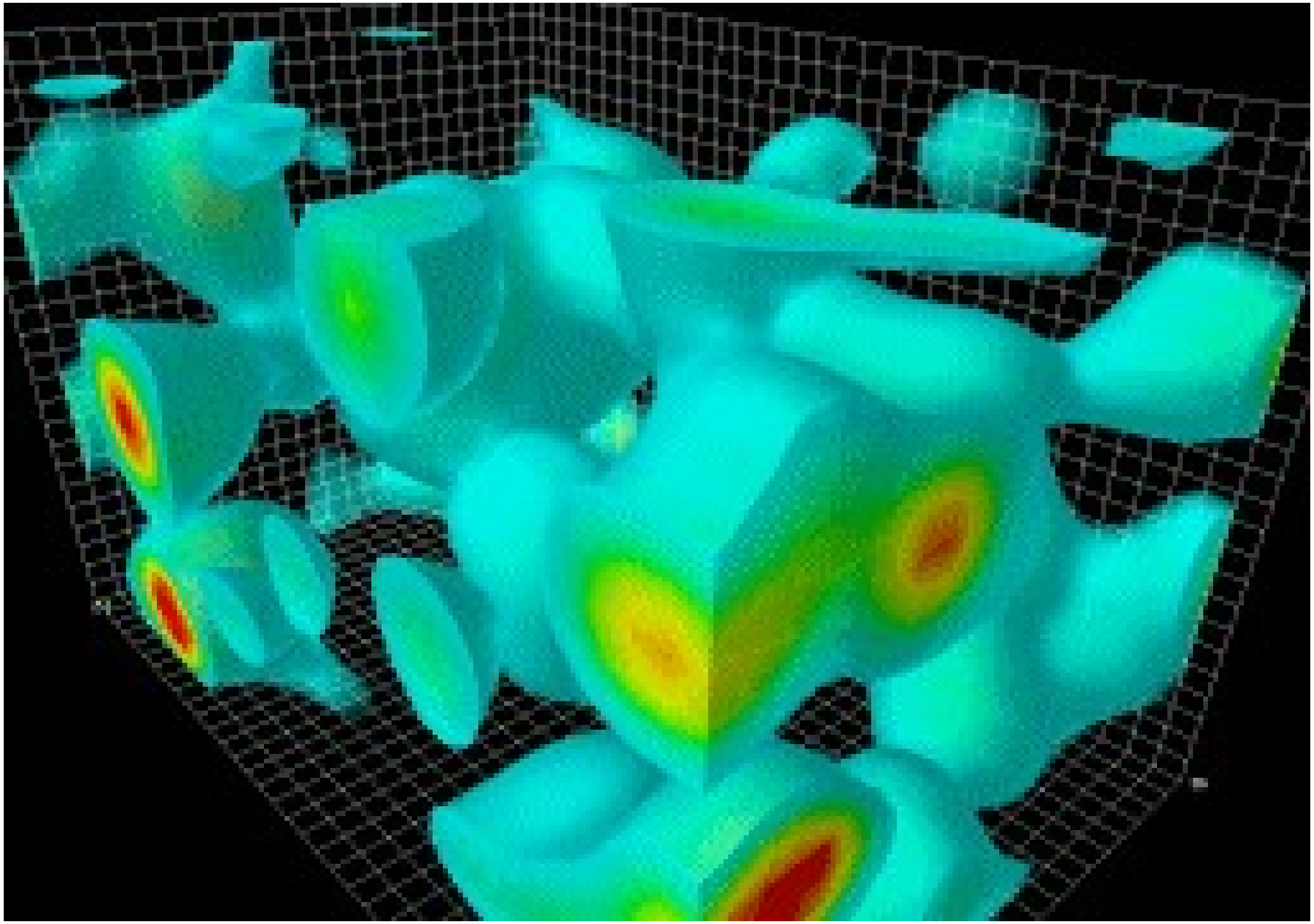
$$A_\mu = A^a_\mu t^a$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\mu - \partial_\nu A^a_\mu + g_{YM} f^{abc} A^b_\mu A^c_\nu$$



$$\mathcal{L}_{YM} \quad = \quad -\frac{1}{4} Tr F^{\mu\nu,\,a} F^a_{\mu\nu}$$





The large N expansion of YM theories

Consider a general theory with some field Φ_i^a in the adjoint of SU(N):

$$\begin{aligned}
\mathcal{L} &\simeq Tr(d\Phi_i d\Phi_i) + g_{YM} c^{ijk} Tr(\Phi_i\Phi_j\Phi_k) + \\
&\quad g_{YM}^2 d^{ijkl} Tr(\Phi_i\Phi_j\Phi_k\Phi_l) \\
\mathcal{L}_{YM} &\simeq Tr(\partial_\mu A_\nu^a \partial_\rho A_\sigma^b) + g_{YM} Tr(\partial_\mu A_\nu^a A_\rho^b A_\sigma^c) + \\
&\quad g_{YM}^2 Tr(A_\nu^a A_\rho^b A_\sigma^c A_\kappa^d)
\end{aligned}$$

Now let us re-scale the fields: $\Phi_i \to \tilde{\Phi}_i \equiv g_{YM}\Phi_i$

$$\begin{aligned}
\mathcal{L} &\simeq \frac{1}{g_{YM}^2} Tr(d\tilde{\Phi}_i d\tilde{\Phi}_i) + \frac{g_{YM}}{g_{YM}^3} c^{ijk} Tr(\tilde{\Phi}_i\tilde{\Phi}_j\tilde{\Phi}_k) + \\
&\quad \frac{g_{YM}^2}{g_{YM}^4} d^{ijkl} Tr(\tilde{\Phi}_i\tilde{\Phi}_j\tilde{\Phi}_k\tilde{\Phi}_l) \\
\tilde{\mathcal{L}} &= \frac{1}{g_{YM}^2}\mathcal{L} = \frac{N}{\lambda}\mathcal{L} \qquad \lambda \equiv g_{YM}^2 N
\end{aligned}$$

Thus, a diagram with

V: vertices (dots)

E: propagators (edges)

F: loops (faces)

in a simplicial decompositions comes proportional to

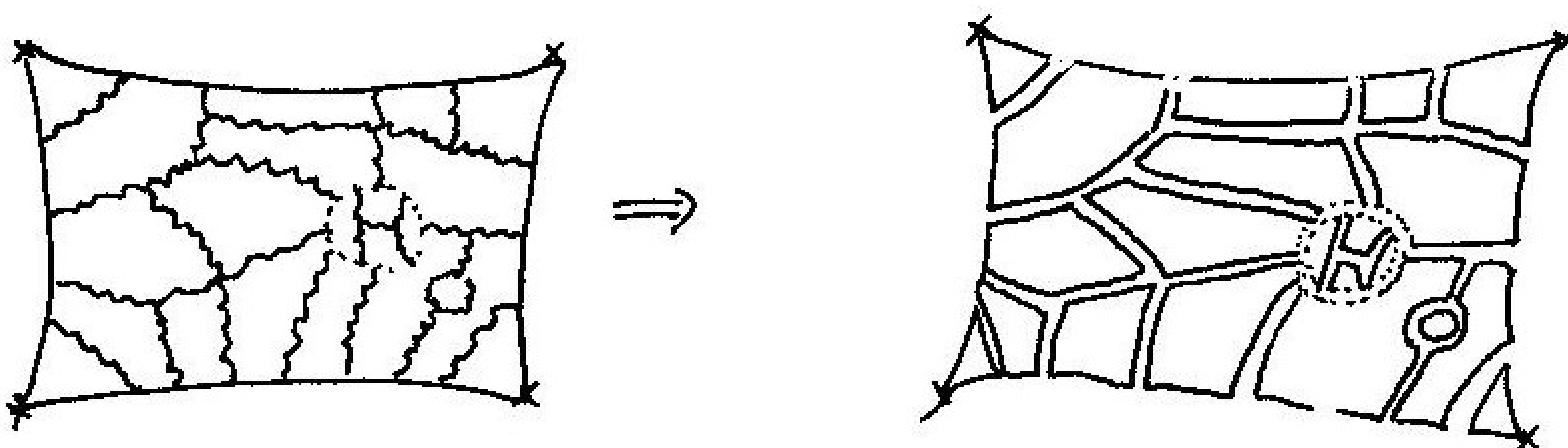
$N^{V-E+F}\lambda^{E-V} = N^{\chi}\lambda^{E-V}$

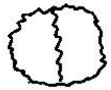
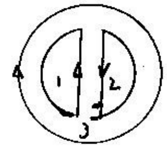
$\chi = V - E + F = 2 - 2g$ the Euler characteristic of the surface. g is the genus (number of handles of the surface)

e.g.

etc.

e.g. 4-point function

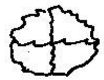
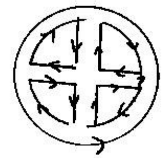


$F=3$
$V=2$
$E=3$

$F+V-E=3+2-3=2$

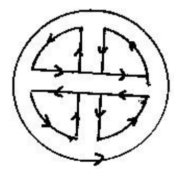
N^2

$F=5$
$V=5$
$E=8$

$5+5-8=2$

N^2

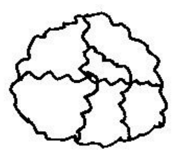
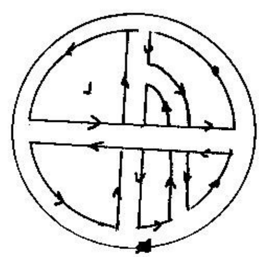
 

$F=2$
$V=4$
$E=6$

$2+4-6=0$

N^0

+1 +2 +3

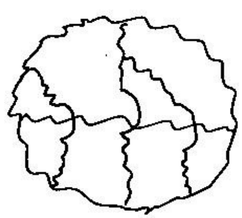
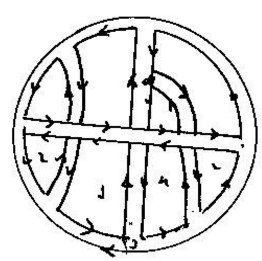
 

$F=3$
$V=6$
$E=9$

$3+6-9=0$

N^0

+1 +2 +3

$F=4$
$V=8$
$E=12$

$4+8-12=0$

N^0

For any diagram there is a perturbative expansion of the form:

$$\sum_{g=0}^{\infty} N^{2-2g} \sum_{i=0}^{\infty} C_{g,i} \lambda^i = \sum_{g=0}^{\infty} N^{2-2g} f_g(\lambda)$$

the surfaces which dominate the expansion are those with minimum genus or maximum Euler characteristic, thus planar diagrams in the large N, the rest will be suppressed by $1/N^2$.

This expansion is the same one finds in a perturbative theory with closed oriented strings, identifying the string coupling g_s with $1/N$.

This is one of the strongest motivations to believe that in the large N limit gauge theory and string theory are dual. This is not a proof obviously.

String theory side.

The dynamics of the strings is described by the Nambu-Goto and the Polyakov actions (the Polyakov's one is)

$$S_{Polyakov} = M_s^2 \int d\tau \int d\sigma (\partial_\tau X_i \partial_\tau X_i - \partial_\sigma X_i \partial_\sigma X_i)$$

It is effectively the action of a QFT in 2D with 24 or 8 scalar massless fields. In the supersymmetric string theory we have to add the fermions, and the critical dimension is 10.

The equation of motion is a wave equation for a free string oscillating in 24 or 8 perpendicular directions:

$$\partial_\tau^2 X_i - \partial_\sigma^2 X_i = 0$$

The theory is conformal.

The quatum theory is obtained in Euclidean space with the Feynman path integral formalism on the Polyakov action

$$Z = \int [dT][dg]/V \exp -S$$

where V is the volume of the groups of Weyl and diffeomorphism invariance and the Euclidean action is

$$S = S_{Polyakov} + X$$

Thus the contribution coming from each Riemann surface is weighted by

$$(1/g_s)^{2-2g-h}$$

Thus identifying $N_c = N \equiv 1/g_s$ one gets the same weight factor as the $SU(N_c)$ gauge theory!

This is crucial for the gauge/string duality.

Comment: Very hard to think a pure fermionic theory satisfying this condition! $\Rightarrow$ why would it to work for AdS/CMT??? No answer by now.

Homework try for instance with a four fermion interaction like Fermi int. It does not work.

QCD, glue and superglue

$$\mathcal{L}_{QCD} = -\frac{1}{2}\,tr\,F^2 + \bar{q}_F^a\,(-i\gamma^\mu\partial_\mu + m_F)\,q_F^a + g_{YM}\,\bar{q}_{Fb}\,\gamma^\mu\,A^b_{\mu a}\,q_F^a$$

$$\alpha_s \equiv \frac{g_{YM}^2}{4\pi}$$

For $N_c = 3 \rightarrow$

$$\beta = -\frac{\left(11 - \frac{2}{3}\right)\ n_F}{(4\pi)^2}\,g_{YM}^3$$

- UV asymptotic freedom.
- IR color confinement at strong coupling.

- Large N_c QFT's: $\lambda \equiv g_{YM}^2 N_c =$ fixed $\longrightarrow$ only planar graphs contribute!

Idea: $SU(3_c) \longrightarrow SU(N_c)$ and for large N_c the dynamics is controlled by the $1/N_c$ expansion.

Consider quark-antiquark system: the separation energy

$E = \sigma\, L$ indicates quark confinement!

Removing the fundamental quarks leads to closed flux tubes: the glueballs.

Glueball has not dynamical quarks $\longrightarrow$ gludynamics.

The large N_c limit is good for studying confinement.

- Numerics: Lattice QCD

Quenched approximation: very heavy quarks.

Pure $\mathcal{N} = 1$ SYM 4d (supergluodynamics)

$$\mathcal{L}_{superglue} = -\frac{1}{2} tr\, F^2 + \bar{\lambda} \left(-i\gamma^\mu D_\mu\right) \lambda$$

$\lambda \equiv$ gauginos.

(Can be straightforwardly extended to SQCD)

The field theory side

The $\mathcal{N} = 4$ SYM Lagrangian (an overall trace is taken)

$$
\begin{aligned}
\mathcal{L} \;=\; & -\frac{1}{2g^2} F^a_{\mu\nu} F^{\mu\nu,a} + \frac{\Theta_I}{8\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu,a} \\
& - \sum_a i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a - \sum_i D_\mu X^i D^\mu X^i \\
& + \sum_{a,b,i} g\, C_i^{ab} \lambda_a [X^i, \lambda_b] + \sum_{a,b,i} g\, \bar{C}_{iab} \bar{\lambda}^a [X^i, \bar{\lambda}^b] \\
& \frac{g^2}{2} \sum_{i,j} [X^i, X^j]^2
\end{aligned}
$$

$\mathcal{N} = 4$ Gauge multiplet (A^a_μ, λ, X_i), with the left Weyl fermions, X^i are 6 real scalars.

Dynamical phases:

Coulomb phase: $< X^i > \neq 0$ for at least one i.
$4 \times 4 \times 1 = 16$ supercharges.

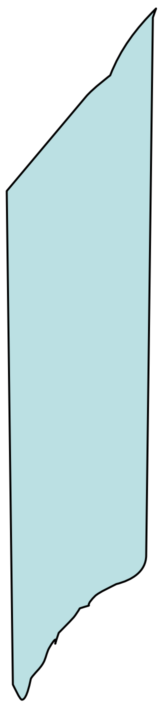
Superconformal phase: $< X^i >= 0$ for all i.
$4 \times 4 \times 2 = 32$ supercharges.

We shall consider the Superconformal Phase of the theory. It has the following symmetries:

- Conformal symmetry: $SO(2,4) \approx SU(2,3)$; generators P^μ; $L_{\mu\nu}$; D, K^μ.
- R-symmetry: $SO(6)_R \approx SU(4)_R$; generators T^A; $A = 1, \cdots, 15 \equiv 4^2 - 1$.
- Poincare supersymmetries: Q^a_α, $\bar{Q}_{\dot{\alpha}a}$, $a = 1, \cdots, 4$; 16 generators.
- Conformal supersymmetries: $S_{\alpha a}$, $\bar{S}^a_{\dot{\alpha}}$, $a = 1, \cdots, 4$; 16 generators.

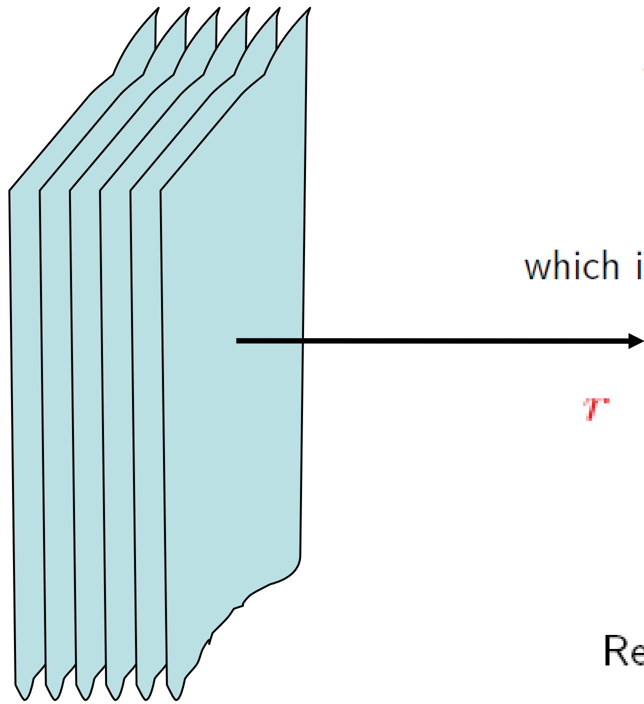
Introduction to the AdS/CFT correspondence:
The Supergravity side

Consider IIB supergravity, a D3-brane in Minkowski 10d = 0 1 2 3 4 5 6 7 8 9



$\boxed{0\ 1\ 2\ 3}$

A D3-brane picture



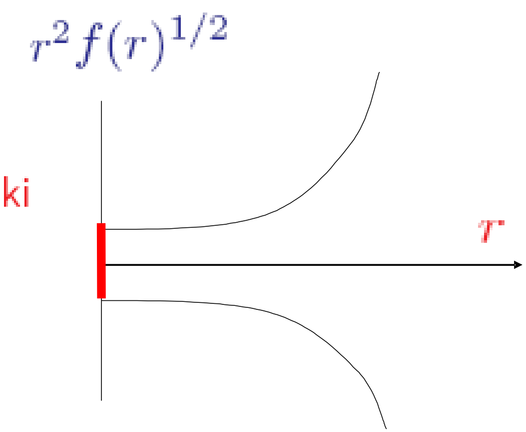
For N coincident D3 branes the induced background metric is

$$
\begin{aligned}
ds^2 &= f^{-1/2}(r)d\vec{x}^2 + f^{1/2}(r)(dr^2 + r^2 d\Omega_5^2) \\
f(r) &= \left(1 + \frac{L^4}{r^4}\right) \\
L^4 &= 4\pi g_s N (\alpha')^2 \qquad \alpha' = l_s^2
\end{aligned}
$$

which is a solution of type IIB supergravity.

Regimes

- $r >> L \rightarrow$ 10d Minkowski
- $r < L \rightarrow$ 10d throat
- $r << L \rightarrow AdS_5 \times S^5$



After changing variables $r \longrightarrow z = \frac{L^2}{r}$ and taking the decoupling limit

$$\lim_{\alpha' \to 0,\ r \to 0} z = \lim_{\alpha' \to 0,\ r \to 0} \frac{L^2}{r} = const$$

the metric becomes $AdS_5 \times S^5$

$$ds^2 = \frac{L^2}{z^2}(d\vec{x}^2 + dz^2 + z^2 d\Omega_5^2)$$

AdS_5 has a boundary which is 4d Minkowski spacetime.

The $\alpha' \to 0$ limit, g_s and N fixed

What happens in this limit?

- Firstly: the limit exists in string theory!
- Only the $\mathrm{AdS}_5 \times S^5$ of the D3-brane system geometry survives and contribute to the dynamics.
- The asymptotically flat region decouples from the theory!

The decoupling limit $\alpha' \to 0$

Inserting the full D3-brane metric scaled as

$G_{MN}(x) \equiv L^2 \overline{G}_{MN}(x; L)$ into the non-linear sigma model

$$
\begin{aligned}
S_\sigma &= \frac{1}{4\pi\alpha'} \int_\Sigma \sqrt{\gamma}\gamma^{mn} G_{MN} \partial_m x^M \partial_n x^N \\
& \quad \frac{L^2}{4\pi\alpha'} \int_\Sigma \sqrt{\gamma}\gamma^{mn} \overline{G}_{MN} \partial_m x^M \partial_n x^N \, .
\end{aligned}
$$

Thus

$$
\frac{L^2}{4\pi\alpha'} = \sqrt{\frac{\lambda}{4\pi}} \qquad\qquad \lambda \equiv g_s N
$$

If we keep g_s and N fixed but take $\alpha' \to 0$ and $L^2 \to 0$ then the scaled metric reduces to $\mathrm{AdS}_5 \times S^5$ and the sigma model becomes

$$
S_\sigma = \sqrt{\frac{\lambda}{4\pi}} \int_\Sigma \sqrt{\gamma}\gamma^{mn} \overline{G}_{MN}(x; L \to 0) \partial_m x^M \partial_n x^N
$$

Thus $1/\sqrt{\lambda}$ has taken the role of α'.

The AdS/CFT conjecture

The large N limit of SU(N) $\mathcal{N} = 4$ SYM theory (superconformal phase) in d=4 is dual to type IIB supergravity on $AdS_5 \times S^5$, with N units of F_5 flux through S^5 and constant dilaton ϕ.

Identification: $e^{-\phi} = g_s \equiv g_{YM}^2$.

The 't Hooft limit

- $\lambda_{'tHooft} \equiv g_{YM}^2 N \equiv g_s N = \lambda$
- with $N \to \infty$
- λ fixed
- therefore g_{YM}, the QFT is perturbative!

The large λ limit

- If $\lambda >> 1$ the QFT is non-perturbative!
- In this region supergravity is a good description.

Identifications I

$$g_s \equiv g_{YM}^2$$

$$L^4 = 4\pi N g_s \alpha' = 4\pi N g_{YM}^2 \alpha' = 4\pi \lambda \alpha'$$

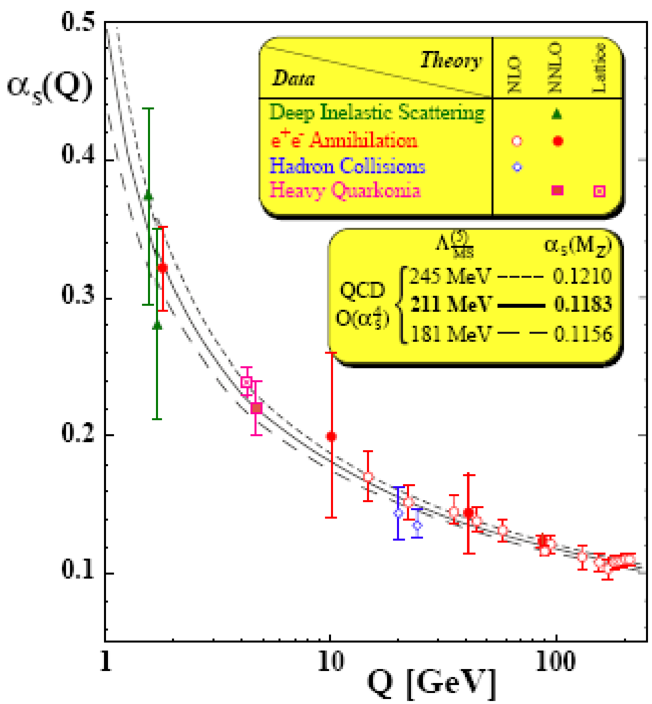
$$\lim_{L\to\infty} curvature \sim 1/L \to 0$$

$$\lim_{L\to\infty} \lambda \to \infty$$

Small curvature

Large t´Hooft coupling= strongly coupled QFT

So, we have a powerful tool to calculate QFT properties at strong coupling!



α_s 2002

S. Bethke[a]

Identifications II

- Mapping global bosonic symmetries: $\mathcal{N} = 4$ SC-phase has $SU(2,2|4)$ as a continous global symmetry.

- Its maximal subgroup is $SU(2,2) \times SU(4)_R \approx SU(2,4) \times SO(6)$, which are the isometries of AdS_5 and S^5, respectively.

- Finally mapping the KK states of type IIB sugra on $AdS_5 \times S^5$ and CFT operators.

Identifications III: calculations of correlators

Mapping SYM and AdS correlators

The ansatz of the precise relation of CFT on the boundary to AdS space is that the QFT generating functional is

$$Z_{QFT}[\phi_0] =< \exp(\int_{bound} dx^4 \mathcal{O}(x^\mu)\phi_0(x^\mu)) >_{CFT} \equiv e^{-I_{SUGRA}}$$

where $\phi_0(x^\mu) = \lim_{z\to 0}\phi(x^\mu, z)$ boundary values of the bulk fields, and $\mathcal{O}(x^\mu)$ are 4d CFT operators. For n-point CFT correlators we have

$$< 0|\hat{T}(\mathcal{O}(x_1^\mu)\cdots\mathcal{O}(x_n^\mu))|0 > \equiv$$

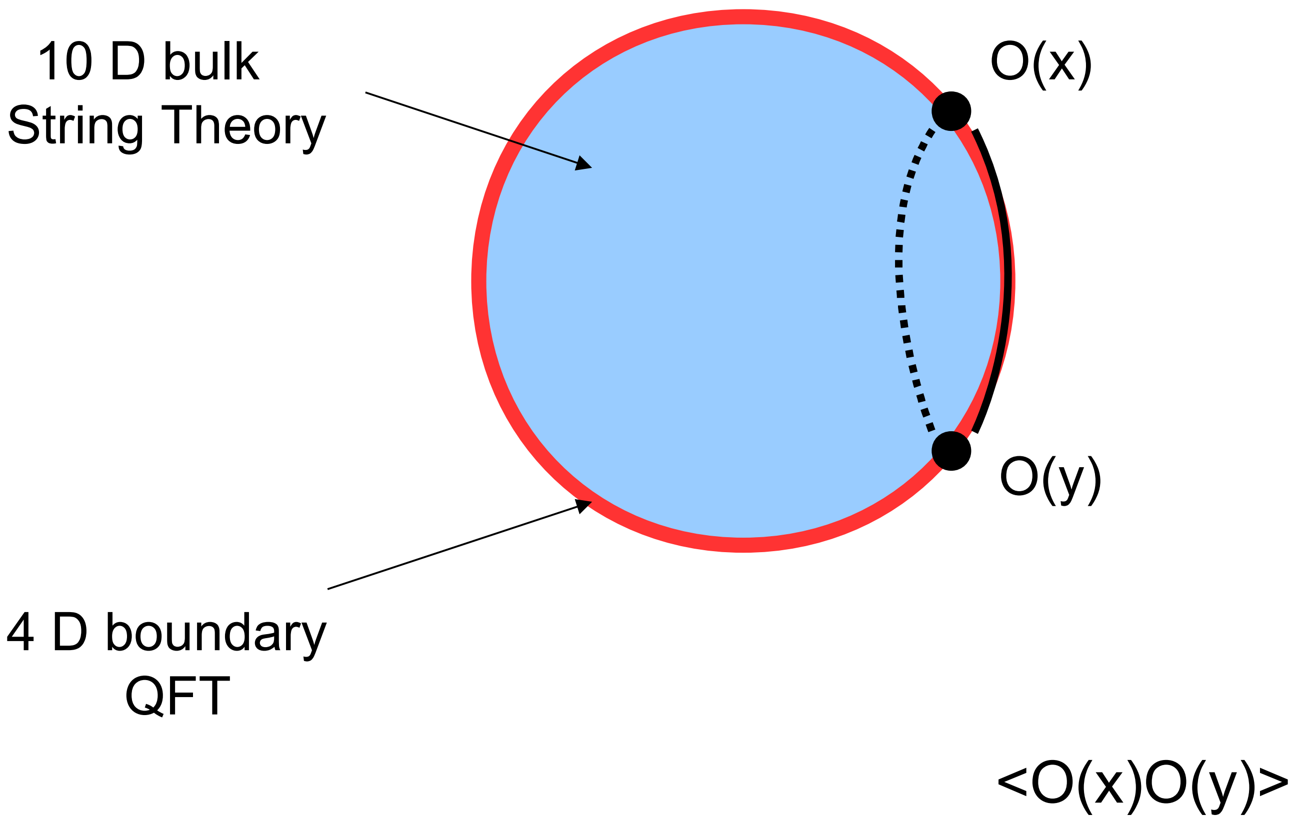
$$\frac{\delta^n < \exp(\int_{boundary}\mathcal{O}(x^\mu)\phi_0(x^\mu)) >_{CFT}}{\delta\phi_0(x_1^\mu)\cdots\delta\phi_0(x_n^\mu)}|_{\phi_0(x^\mu)\to 0} =$$

$$\frac{\delta^n Z_S[\phi_0(x^\mu)]}{\delta\phi_0(x_1^\mu)\cdots\delta\phi_0(x_n^\mu)}|_{\phi_0(x^\mu)\to 0}$$

Important point:

Notice that: $e^{\phi} = g_s = g_{YM}^2$ is a constant in a CFT $\longrightarrow \phi =$ constant $\longrightarrow$ AdS is FIXED POINT in a sense that will be clarified now.

AdS/CFT: The Idea



UV/IR connection

$$
\begin{aligned}
ds^2_{10D} &= ds^2_{AdS_5} + R^2 d\Omega_5^2 \\
ds^2_{AdS_5} &= \frac{r^2}{R^2}\eta_{\mu\nu}dx^\mu dx^\nu + \frac{R^2}{r^2}dr^2
\end{aligned}
$$

r from 0 to infinity, Poincare patch of a global AdS spacetime. Now using $z \equiv R^2/r$

$$
\begin{aligned}
ds^2_{AdS_5} &= \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) \\
d &= \frac{R}{z}d_{YM} \qquad E = \frac{z}{R}E_{YM}
\end{aligned}
$$

$z = 0$ AdS boundary. $z \to \infty$ Poincare horizon.

UV $E_{YM} \to \infty$ corresponds to $z \to 0$ (AdS boundary).

IR $E_{YM} \to 0$ corresponds to $z \to \infty$ (AdS horizon).

Finite T

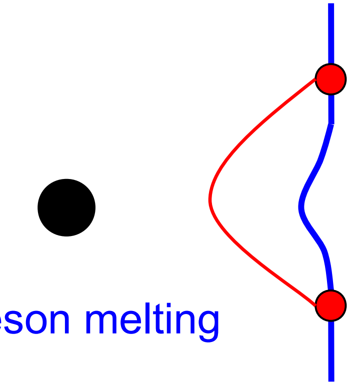
- $\mathcal{N} = 4$ SYM for $T = 0 \Leftrightarrow AdS_5 \times S^5$ in type IIB string theory.

- $\mathcal{N} = 4$ SYM for $T > 0 \Leftrightarrow AdS_5$ Schwarzschild BH $\times S^5$ in type IIB string theory.
- It describes glueball-to-gluon deconfinement: Glueball Plasma

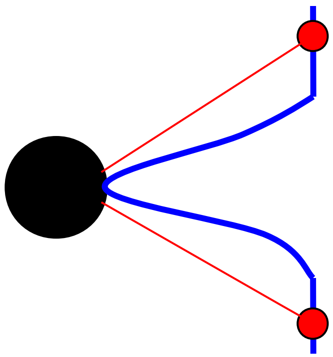
Finite T gauge theories

Flavours using brane probes. Small number of D7-branes probing $AdS_5 \times S^5$: Chiral symmetry breaking, 4d completion at UV.

Meson spectra at finite T from fluctuations of the D7-brane.



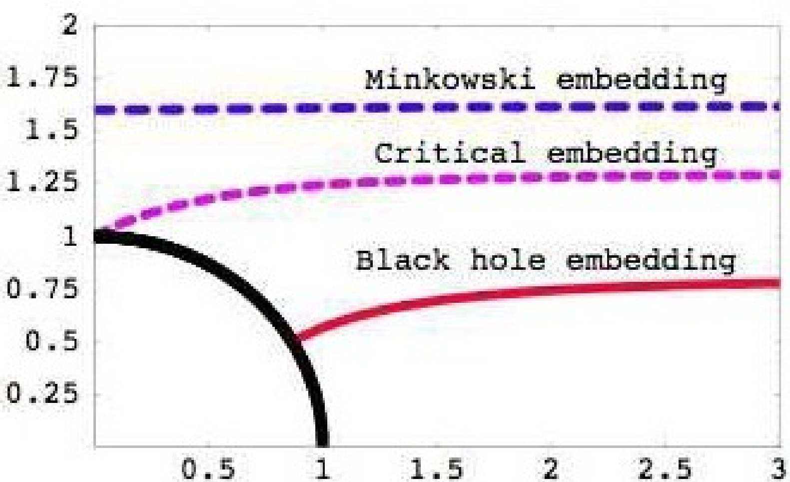
glueball deconfinement < T < meson melting



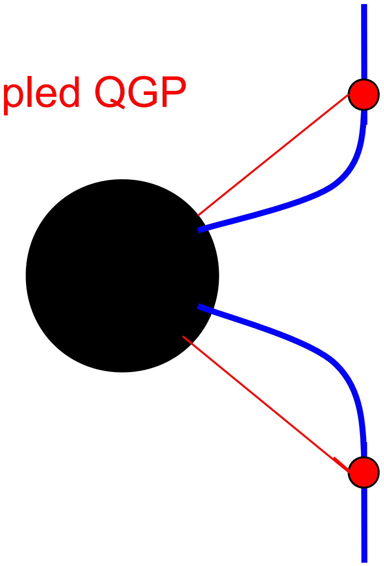
Critical embedding: T = meson melting

Vector meson melting, using the black hole embedding.

Studies of strongly coupled QGP.



meson melting = strongly coupled QGP



Green-Kubo formula for transport coefficients

Let us consider the system in the rest frame $u^\mu = (1,0)$. Deviations from the equilibrium are studied by introducing a small perturbation (as an external source)

$$S = S_0 + \frac{1}{2}\int d^4x T^{\mu\nu} h_{\mu\nu}$$

To the leading order:

$$< T^{\mu\nu}(x) > = < T^{\mu\nu}(x) >_0 - \frac{1}{2}\int d^4x G_R^{\mu\nu,\alpha\beta}(x-y) h_{\alpha\beta}(y)$$

$$iG_R^{\mu\nu,\alpha\beta}(x-y) \equiv \Theta(x^0 - y^0) < [T^{\mu\nu}(x), T^{\alpha\beta}(y)] >$$

to extract the shear viscosity we concentrate on an external perturbation:

$$< T^{xy}(\omega, k) > = -G_R^{xy,xy}(\omega,k) h_{xy}(\omega,k)$$

$$< T^{xy} > (t,z) = -\int \frac{d\omega}{4\pi} e^{-i\omega t} G_R^{xy,xy}(\omega, k=0) h_{xy}(\omega, z)$$

having considered the long wavelength limit.

Green-Kubo formula for transport coefficients

This long wavelength expression may be compared to the hydrodynamic approximation by studying the reaction of the system to a source in the effective theory, which can be interpreted as a fluctuation of the metric:

$$g_{\mu\nu} \to g_{\mu\nu} + h_{\mu\nu}$$

To the leading order in perturbation theory the shear tensor is defined in terms of the Christoffel symbols:

$$\sigma_{xy} = 2\Gamma^0_{xy} = \partial_0 h_{xy}$$

$$\eta = -\lim_{\omega\to 0}\frac{1}{\omega}\lim_{k\to 0} ImG_R^{xy,xy}(\omega, k)$$

which is the Kubo-Green formula for the shear viscosity. This can be generalised to other currents just by replacing $T^{\mu\nu}$ by any current operator.

Absorption of scalars by black 3-branes and the shear viscosity

Consider a black 3-brane in type IIB supergravity. We want to study what happens to a graviton polarized in the directions (x, y) which are parallel to the brane. The absorption cross-section reads:

$$\sigma(\omega) = \frac{\kappa^2}{\omega} \int dt \int d^3x \, e^{i\omega t} < [T_{xy}(x) T_{xy}(0)] >$$

where $\kappa^2 = 8\pi G_{10}$. Now, by comparison with the Green-Kubo formula we obtain:

$$\eta = \frac{1}{2\kappa^2} \sigma(0)$$

Absorption of scalars by black 3-branes and the shear viscosity

Let us see how to get the absorption cross-section of gravitons by black 3-branes. The metric fluctuation $h_{xy} \equiv \varphi$ is an scalar fluctuation. The metric of N black 3-branes is

$$
\begin{aligned}
ds^2 &= H^{-1/2}(r)(-f(r)dt^2 + d\vec{x}^2) \\
&\quad + H^{1/2}(r)(f^{-1}(r)dr^2 + r^2 d\Omega_5^2) \\
&\quad H(r) = 1 + \frac{R^4}{r^4} \qquad f(r) = 1 - \frac{r_0^4}{r^4} \\
&\quad T = \frac{r_0}{\pi R^2}
\end{aligned}
$$

The EOM for the fluctuation is $\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi) = 0$. Using the Ansatz: $\varphi = e^{i\omega t}\phi(r)$ the EOM becomes

$$
\phi'' + \frac{5r^4(r^4 - r_0^4)}{r(r^4 - r_0^4)}\phi' + \omega^2 \frac{r^4(r^4 + R^4)}{(r^4 - r_0^4)^2}\phi = 0
$$

since $\omega \to 0$ then we consider $\omega << T$. To solve the EOM there are 3 regions. Finally one gets: $\sigma = \pi^3 r_0^3 R^2$.

The Bekenstein-Hawing area law

$$S = \frac{A}{4G}$$

$$S = \frac{A}{4G}\frac{k_B c^3}{\hbar}$$

The area of the even horizon of the 10d BH can be obtained from:

$$ds^2 = H^{-1/2}(d\vec{x}^2) + H^{1/2} r_0^2 d\Omega_5^2$$

$$H = 1 + \frac{R^4}{r^4} \to \frac{R^4}{r^4}$$

$$V_8 = \int \sqrt{g} d^8 x = \pi^3 R^2 r_0^3 V_3$$

Therefore we can define $a_H = V_8/V_3 = \pi^3 R^2 r_0^3$ thus the entropy per unit volume is $s = S/V_3 = (V_8/V_3) 1/(4G) = \pi^3 R^2 r_0^3$. Recall that $\sigma(0) = \pi^3 R^2 r_0^3$, and $\eta = 1/(16\pi G)\sigma(0)$, then

$$\frac{\eta}{s} = \frac{1}{16\pi G}\sigma(0)\frac{4G}{\pi^3 R^2 r_0^3} = \frac{1}{16\pi G}\pi^3 R^2 r_0^3 \frac{4G}{\pi^3 R^2 r_0^3} = \frac{1}{4\pi} \simeq 0.08$$

Recall lattice results: for $T = 1.24 T_c$ $\eta/s = 0.102(56)$, H. Meyer.

Finite 't Hooft coupling and N corrections to η/s

$$\frac{\eta}{s} = \frac{1}{4\pi}\left(1 + \frac{15\zeta(3)}{\lambda^{3/2}} + \frac{5}{16}\frac{\lambda^{1/2}}{N^2} + \frac{15}{2\pi^{1/2}N^{3/2}}e^{-8\pi^2 N/\lambda}\right)$$

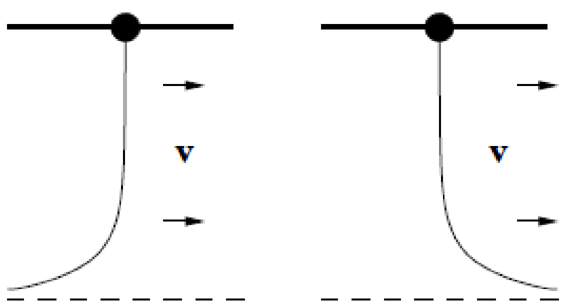
for $\lambda = 6\pi$ which corresponds to $\alpha_2 = g_{YM}^2/(4\pi) = 0.5$ and $N = 3$, this gives 0.08 to be corrected to 0.11 (lattice was 0.102(56)). Corrections from the first term are 22%, then 15% and the third 10^{-7}.

Parton energy loss via a drag on heavy quarks

$x_1(\tau,\sigma)\,,\qquad x_1(t, z\to 0) = vt\,.\qquad x_1(t,z) = vt + \zeta(z)\,,$

$$ds^2_{ws} = \frac{R^2}{z^2}\Bigg(-\left(f(z)-v^2\right)d\tau^2 + \left(\frac{1}{f(z)} + \zeta'^2(z)\right)d\sigma^2 + v\,\zeta'(z)v\,(d\tau\, d\sigma + d\sigma\, d\tau)\Bigg)\,,$$

$$S = -\frac{R^2}{2\pi\alpha'}\mathcal{T}\int\frac{dz}{z^2}\sqrt{\frac{f(z)-v^2+f(z)^2\zeta'^2(z)}{f(z)}} = \mathcal{T}\int dz\,\mathcal{L}\,,$$



$$\Pi^1_z = \frac{\partial\mathcal{L}}{\partial x_1'} = -\frac{R^2}{2\pi\alpha'}\frac{1}{z^2}\frac{f(z)^{3/2}\zeta'(z)}{\sqrt{f(z)-v^2+f(z)^2\zeta'^2(z)}}\,,$$

$$\Pi^1_z = \pm\frac{R^2}{2\pi\alpha' z_0^2}\gamma v\,,$$

$$\frac{dp}{dt} = -\Pi^1_z = -\frac{\pi T^2\sqrt{\lambda}}{2}\gamma v\,.$$

$$\frac{dp}{dt} = -\eta_D p\,,\qquad \eta_D = \frac{\pi\sqrt{\lambda}T^2}{2M}\,.$$

The RHIC (last few years) & LHC (2010-2011) results

- Deconfined quark-gluon plasma (QGP) is produced from heavy ion collisions.
- Data suggest it corresponds to a strongly coupled regime of QCD.
- Plasma behaves like an ideal fluid.
- Excellent environment to apply the gauge/string duality at full extent: supergravity $+$ string corrections.

Results DIS q>>T

$$F_T \equiv 2x_B F_1, \text{ where } x_B = Q^2/(2\omega T)$$

$$F_L \equiv F_2 - 2x_B F_1$$

$$F_1 \simeq \left(1 + \frac{5}{8}\xi(3)\lambda^{-3/2}\right) \frac{3N^2T^2}{16\Gamma^2(1/3)} \left(\frac{q}{6\pi T}\right)^{2/3}$$

$$F_L \simeq \left(1 + \frac{325}{32}\xi(3)\lambda^{-3/2}\right) \frac{N^2Q^2x_B}{96\pi^2}$$

$$F_T \propto \left(1 + \frac{5}{8}\xi(3)\lambda^{-3/2}\right) \frac{N^2T^2}{x_B} \left(\frac{x_B^2Q^2}{T^2}\right)^{2/3}$$

Motivation III: QGP Hydrodynamics: $E < T$

The transport coefficients.

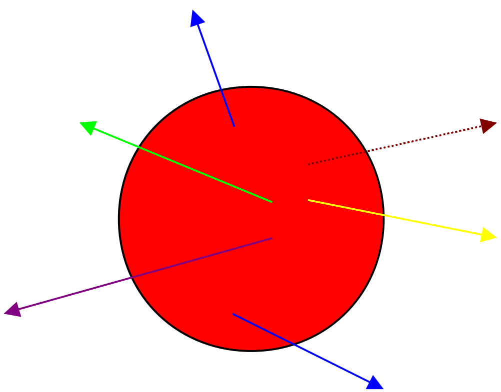
- Mass transport: shear viscosity and diffusion constant.
- Charge transport: conductivity and charge diffusion.

Plasma conductivity large N and large λ

$$\sigma_{QGP} = \lim_{q^0 \to 0} \frac{e^2}{6T} \eta^{\mu\nu} \int d^4x \, e^{-iq\cdot x} \, \Theta(x_0) \, < [J_\mu(x), J_\nu(0)] > |_{\vec{q}=0}$$

Motivation IV: Plasma photo-production

- Plasma in thermal equilibrium, but optically thin.
- Photons are emitted from the plasma, which does not include prompt photons produced by the initial scattering of partons from the colliding nuclei.
- The electromagnetic coupling constant, e, is consider small enough to ensure photons are not to be re-scattered and consequently do not thermalise.



The Wightman function of electromagnetic currents is defined as

$$C^{<}_{\mu\nu}(K) = \int d^4X \, e^{-iK\cdot X} \, < J^{\rm em}_{\mu}(0) J^{\rm em}_{\nu}(X) >$$

In thermal equilibrium is related to the spectral density

$$C^{<}_{\mu\nu}(K) = n_b(k^0) \, \chi_{\mu\nu}(K)$$

Bose-Einstein distribution function $n_b(k^0) = 1/(e^{\beta k^0} - 1)$.

The spectral density is given by the imaginary part of the retarded current-current correlation function

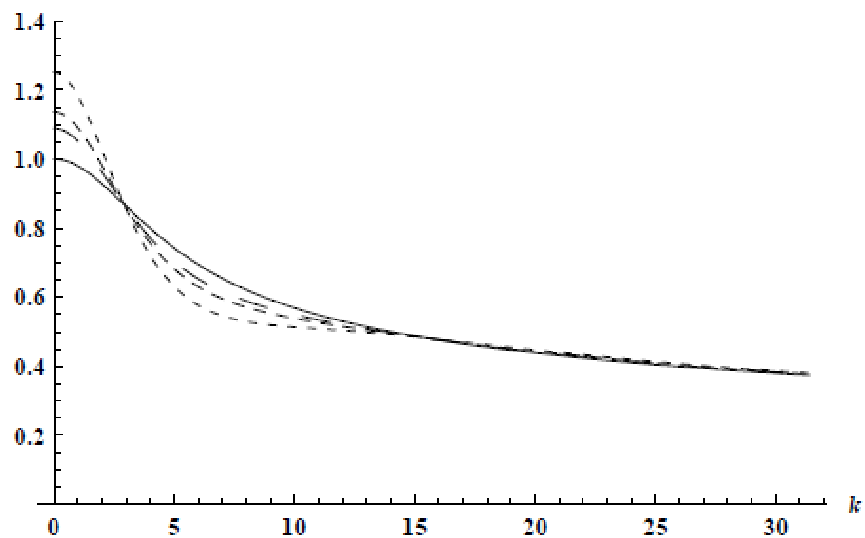
$$\chi_{\mu\nu}(K) = -2\,\mathrm{Im}\, C^{\mathrm{ret}}_{\mu\nu}(K)$$

The number of photons which are produced per unit time per unit volume is denoted by Γ_γ. At leading order in e the photoemission rate is given by

$$d\Gamma_\gamma = \frac{e^2}{2|\vec{k}|}\,\eta^{\mu\nu}\, C^{<}_{\mu\nu}(K)|_{k^0=|\vec{k}|}\,\frac{d^3k}{(2\pi)^3}$$

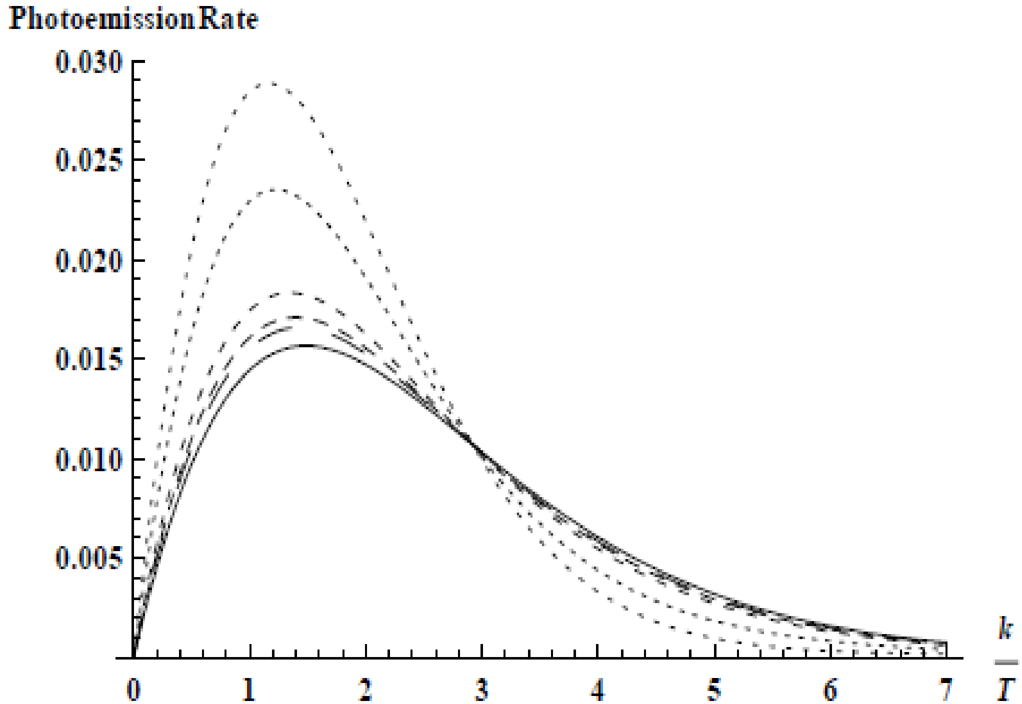
This formula for the photoemission rate holds to leading order in the electromagnetic coupling e, but it is valid non-perturbatively in all other interactions, $i.e.$ strong interaction.

$\lambda = \infty, 200, 150,$ and 100



For very large λ the photoemission rate is given by

$$\frac{d\Gamma_\gamma}{dk} = \frac{\alpha_{\rm em} N^2 T^3}{16\pi^2}\,\frac{(k/T)^2}{e^{k/T}-1}\, \left|\, {}_2F_1\left(1-\frac{(1+i)k}{4\pi T},\ 1+\frac{(1-i)k}{4\pi T};\ 1-\frac{ik}{2\pi T};\ -1\right)\right|^{-2}$$

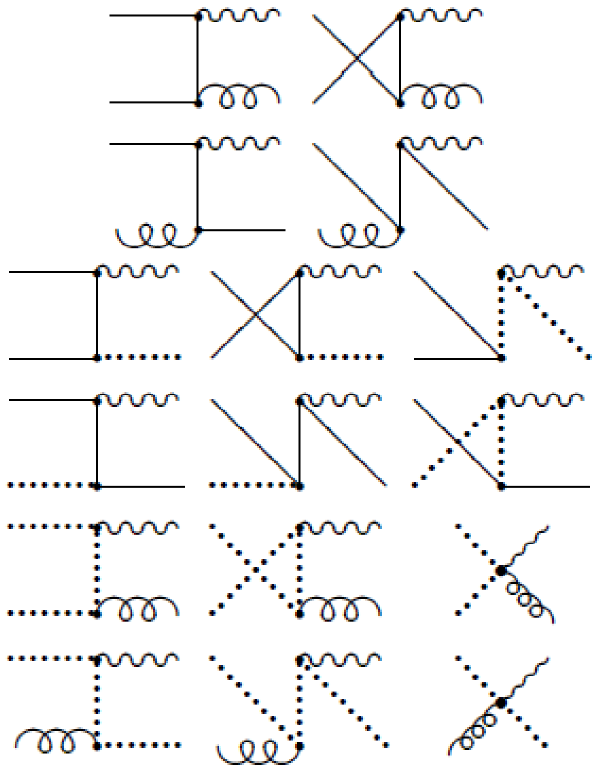


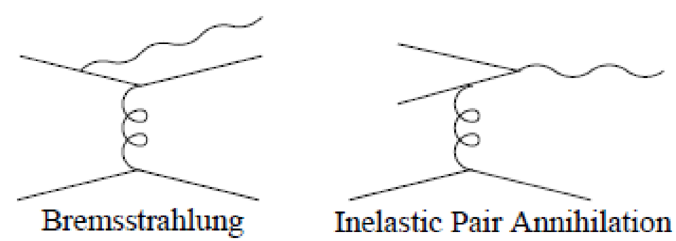
Perturbatively:

$$\frac{d\Gamma_\gamma}{dk} = \frac{(N^2-1)\alpha_{\rm em}}{4\pi^2} k\, n_f(k)\, m_\infty^2 \left[\ln(T/m_\infty) + C_{\rm tot}(k/T)\right]$$

$$m_\infty^2 = \lambda T^2$$

$$C_{\rm tot}(k/T) = \frac{1}{2}\ln(2k/T)$$
$$+C_{2\longleftrightarrow 2}(k/T) + C_{\rm brem}(k/T) + C_{\rm pair}(k/T)$$





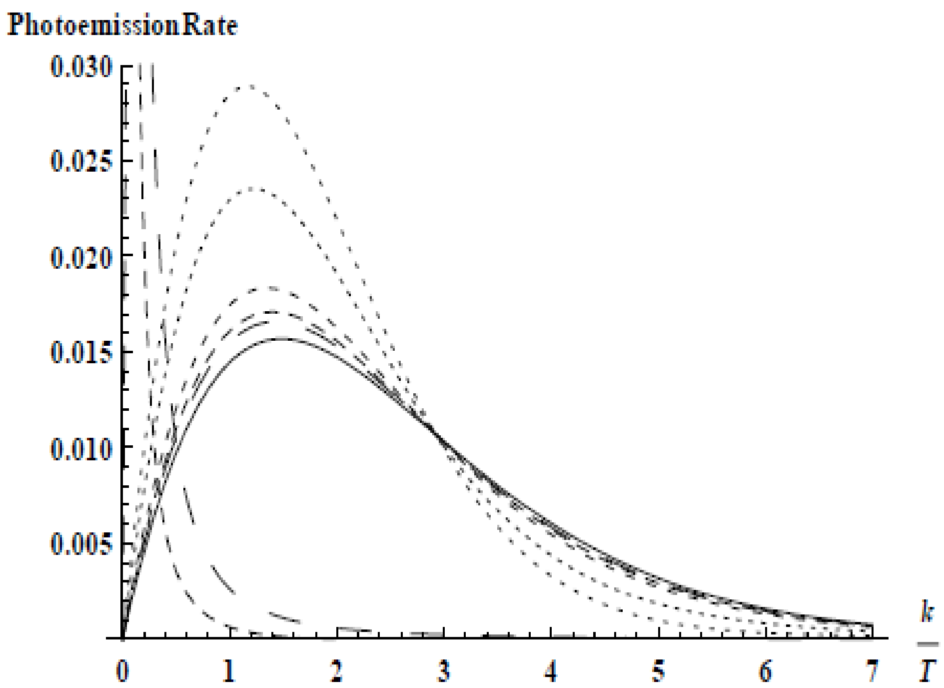
Bremsstrahlung
Inelastic Pair Annihilation

$$C_{2\longleftrightarrow 2}(k/T) \simeq 2.01\,T/k - 0.158 - 0.615\,e^{-0.187k/T}$$

$$C_{\mathrm{brem}}(k/T) + C_{\mathrm{pair}}(k/T) \simeq$$

$$0.954\,(T/k)^{3/2}\,\ln(2.36 + T/k) + 0.069 + 0.0289\,k/T\,,$$

which hold in the range $0.2 < k/T < 20$.



Important conceptual and technical developments

The AdS/CFT conjecture

The large N limit of SU(N) $\mathcal{N} = 4$ SYM theory (superconformal phase) in d=4 is dual to type IIB supergravity on $AdS_5 \times S^5$, with N units of F_5 flux through S^5 and constant dilaton ϕ.

Identification: $e^{-\phi} = g_s \equiv g_{YM}^2$.

What about the QFT?

The $\mathcal{N} = 4$ SYM Lagrangian (an overall trace is taken)

$$\begin{aligned} \mathcal{L} &= -\frac{1}{2g^2} F^a_{\mu\nu} F^{\mu\nu,a} + \frac{\Theta_I}{8\pi^2} F^a_{\mu\nu} \tilde{F}^{\mu\nu,a} \\ & -\sum_a i\bar{\lambda}^a \bar{\sigma}^\mu D_\mu \lambda^a - \sum_i D_\mu X^i D^\mu X^i \\ & +\sum_{a,b,i} g\, C_i^{ab} \lambda_a [X^i, \lambda_b] + \sum_{a,b,i} g\, \bar{C}_{iab} \bar{\lambda}^a [X^i, \bar{\lambda}^b] \\ & \frac{g^2}{2} \sum_{i,j} [X^i, X^j]^2 \end{aligned}$$

The C's are related to the Clifford Dirac matrices for $SO(6)_R \approx SU(4)_R$.

$\mathcal{N} = 4$ Gauge multiplet (A^a_μ, λ, X_i), with the left Weyl fermions, X^i are 6 real scalars.

Finite temperature description

The AdS_5-Schwarzschild black hole $\times S^5$

$$ds^2 = \left(\frac{r_0}{L}\right)^2 \frac{1}{u}\left(-f(u)dt^2 + d\vec{x}^2\right) + \frac{L^2}{4u^2 f(u)}du^2 + L^2 d\Omega_5^2\,,$$

where $f(u) = 1 - u^2$ and the b.h. temperature: $r_0 = \pi T L^2$.

So, the conjecture it that a deconfined QGP is described by this supergravity solution.

The type IIB supergravity action

$$S_{10} = \frac{1}{2\kappa_{10}^2} \int \ d^{10}x \sqrt{-G} \left[R_{10} - \frac{1}{2} (\partial\phi)^2 - \frac{1}{4.5!} (F_5)^2 \right]$$

Leading string theory corrections

$$S_{10}^{\alpha'} = \frac{L^6}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G} \left[\gamma e^{-\frac{3}{2}\tilde{\phi}} W_4 + \cdots \right]$$

- $\gamma \equiv \frac{1}{8}\,\xi(3)\,(\alpha'/L^2)^3$
- $L^4 = 4\pi g_s N \alpha'^2$
- Setting $\lambda = g_{YM}^2 N \equiv 4\pi g_s N$
- $\gamma = \frac{1}{8}\,\xi(3)\,\frac{1}{\lambda^{3/2}}$.

The W_4 term is a dimension-eight operator, and is given by

$$W_4 = C^{hmnk}\, C_{pmnq}\, C_h{}^{rsp}\, C^q{}_{rsk} + \frac{1}{2} C^{hkmn}\, C_{pqmn}\, C_h{}^{rsp}\, C^q{}_{rsk}\,,$$

$C^q{}_{rsk}$ is the Weyl tensor.

Dots denote extra corrections containing contractions of F_5, schematically:

$$\gamma(C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4)$$

General compact 5d Einstein manifolds

- $R_{ab} = \mathcal{R}/d\, g_{ab}$

  Includes:

- Sasakian manifold M is one with the Riemannian cone Kähler. If the cone is Ricci-flat, M is called Sasaki-Einstein, in 5d $L_{p,q,r}$.

- Examples include all round odd-dimensional spheres, and the product of a 2-sphere and a 3-sphere with a homogeneous metric. The cones are respectively complex vector spaces without the origin, and the conifold $T^{1,1}$, Klebanov-Witten dual.

- Some circle bundles over the 3rd through 8th del Pezzo surfaces.

- $Y^{p,q}$, quiver theories.

General case

$$
\begin{aligned}
ds^2 &= \left(\frac{r_0}{L}\right)^2 \frac{1}{u} \left(-f(u)\, K^2(u)\, dt^2 + d\vec{x}^2\right) \\
&\quad + \frac{L^2}{4u^2 f(u)}\, P^2(u)\, du^2 + L^2 R^2(u)\, dM_5^2 \\
K(u) &= \exp\left[\gamma\,(a(u) + 4b(u))\right], \quad P(u) = \exp\left[\gamma\, b(u)\right] \\
R(u) &= \exp\left[\gamma\, c(u)\right] \\
a(u) &= -\frac{1625}{8}\, u^2 - 175\, u^4 + \frac{10005}{16}\, u^6 \\
b(u) &= \frac{325}{8}\, u^2 + \frac{1075}{32}\, u^4 - \frac{4835}{32}\, u^6\,, \\
c(u) &= \frac{15}{32}\,(1+u^2)\, u^4\,, \qquad \text{with} \qquad r_0 = \frac{\pi T R^2}{(1 + \frac{265}{16}\gamma)}
\end{aligned}
$$

The simplest example $M_5 = S^5$

$$
\begin{aligned}
ds^2 &= g_{mn}\, dx^m\, dx^n \\
&\quad + L^2 R(u)^2 \sum_{i=1}^{3} \left[d\mu_i^2 + \mu_i^2 (d\phi_i + \frac{2}{\sqrt{3}} A_\mu dx^\mu)^2 \right] \\
F_5 &= G_5 + *G_5 \\
G_5 &= -\frac{4}{R}\bar{\epsilon}_5 + \frac{R^3 L(u)^3}{\sqrt{3}} \left(\sum_{i=1}^{3} d\mu_i^2 \wedge d\phi_i \right) \wedge \bar{*} F_2
\end{aligned}
$$

where $F_2 = dA$ is the Abelian field strength and

$$
S = -\frac{\tilde{N}^2}{64\pi^2 L} \int d^4x\, du\, \sqrt{-g}\, R^7(u)\, g^{mp}\, g^{nq}\, F_{mn}\, F_{pq}
$$

Operator enumeration for the general case

AdS_5 Schwarzschild black hole with and Einstein manifold. The ten-dimensional corrections containing the five-form field strength are schematically

$$C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4$$

$$\begin{aligned}
\mathcal{T}_{abcdef} &= i\nabla_a F^+_{bcdef} + \frac{1}{16}\left(F^+_{abcmn}F^{+\ mn}_{def} - F^+_{abfmn}F^{+\ mn}_{dec}\right) \\
F^+ &= \frac{1}{2}(1+*)F_5
\end{aligned}$$

The self-duality constraint

$$F_5 - 120\gamma\frac{\delta W_{\mathcal{R}^4}}{\delta F_5} = *\left(F_5 - 120\gamma\frac{\delta W_{\mathcal{R}^4}}{\delta F_5}\right)$$

$$
\begin{aligned}
&(C^3\mathcal{T}+C^2\mathcal{T}^2+C\mathcal{T}^3)|_{5d} = \\
&a_1 C_{abcd}C^{abcd}\nabla^e F_{ef}\nabla^g F^f{}_g + a_2 C_{abcd}C^{acbd}\nabla_e F_{fg}\nabla^f F^{eg} + \\
&C_{abc}{}^d C^{acbe}\left[b_1\nabla_f F_d{}^f\nabla_g F_e{}^g +_2 \nabla_d F_{ef}\nabla_g F^{fg} + b_3\nabla_d F_{fg}\nabla_e F^{fg}\right] + \\
&b_4 C_{abc}{}^d C^{abce}\nabla_f F_{dg}\nabla^f F_e{}^g \\
&C_a{}^b{}_c{}^d C^{aecf}\left[c_1\nabla_b F_{eg}\nabla^g F_{df} + c_2\nabla_b F_{de}\nabla^g F_{fg} + c_3\nabla_b F_{dg}\nabla_f F_e{}^g + c_4\nabla_b F_{eg}\nabla_d F_f{}^g\right] + \\
&C_a{}^b{}_c{}^d C^{acef}\left[c_5\nabla_b F_{dg}\nabla_e F_f{}^g + c_6\nabla_b F_{ef}\nabla_g F_d{}^g + c_7\nabla_b F_{eg}\nabla_f F_d{}^g\right] \\
&+c_8 C_{ab}{}^{cd}C^{abef}\nabla_c F_{eg}\nabla^g F_{df} + \\
&C_a{}^{bcd}C^{aefg}\left[d_1\nabla_b F_{ce}\nabla_f F_{dg} + d_2\nabla_c F_{be}\nabla_f F_{dg} + d_3\nabla_b F_{cf}\nabla_g F_{de} + d_4\nabla_c F_{bd}\nabla_f F_{eg}\right] + \\
&e_1 C_{abcd}C^{acbd}F_{ef}F^{ef} + f_1 C_{abc}{}^d C^{acbe}F_{df}F_e{}^f \\
&+g_1 C_a{}^b{}_c{}^d C^{aecf}F_{be}F_{df} + C_a{}^b{}_c{}^d C^{acef}\left[g_2 F_{bd}F_{ef} + g_3 F_{be}F_{df}\right] + \\
&h_1 F_{ab}F^{ab} + h_2\nabla_a F_{bc}\nabla^b F^{ac} + h_3\nabla^b F_{bc}\nabla^a F_a{}^c\,.
\end{aligned}
$$

The Lagrangian for the transverse modes

Inserting the perturbed metric and F_5 into the 5d operators, the minimal kinetic term yields the following Lagrangian for the transverse mode A_x

$$\begin{aligned}
S_{\text{total}} \quad = \quad & -\frac{\tilde{N}^2 r_0^2}{16\pi^2 R^4} \int \frac{d^4k}{(2\pi)^4} \int_0^1 du \; [\gamma A_W A_k'' A_{-k} + \\
& (B_1 + \gamma B_W) A_k' A_{-k}' \\
& + \gamma C_W A_k' A_{-k} + (D_1 + \gamma D_W) A_k A_{-k} \\
& + \gamma E_W A_k'' A_{-k}'' + \gamma F_W A_k'' A_{-k}']
\end{aligned}$$

where Fourier transform of the field A_x

$$A_x(t, \vec{x}, u) = \int \frac{d^4k}{(2\pi)^4} \, e^{-i\omega t + iqz} \, A_k(u) \, .$$

$$
\begin{aligned}
A_W &= -2u^5\left[a_1^w f(u)\kappa_0^2 + a_2^w \varpi_0^2\right] + \tilde{A}_W \,, \\
B_W &= -4u^4\left[(b_1^w - b_2^w u^2 + b_3^w u^4) - b_5^w \varpi_0^2 u - b_4^w u f(u)\kappa_0^2\right] + \tilde{B}_W \,, \\
C_W &= -4\frac{u^4}{f(u)}\left[3f(u)(c_1^w u^2 - c_2^w)\kappa_0^2 + \left(c_4^w - c_3^w u^2\right)\varpi_0^2\right] + \tilde{C}_W \,, \\
D_W &= -\frac{u^3}{f^2(u)}\Big[d_6^w u f^2(u)\kappa_0^4 + d_7^w u \varpi_0^4 + d_8^w u f(u)\kappa_0^2\varpi_0^2 \\
&\quad +4f^2(u)(d_1^w u^2 - d_2^w)\kappa_0^2 + 4(d_3^w - d_4^w u^2 + d_5^w u^4)\varpi_0^2\Big] + \tilde{D}_W \,, \\
E_W &= -e_1^w u^6 f^2(u) + \tilde{E}_W \,, \\
F_W &= 4u^5 f(u)(f_1^w u^2 - f_2^w) + \tilde{F}_W \,,
\end{aligned}
$$

$$
\begin{aligned}
\tilde{A}_W &= 0\,, \\
\tilde{B}_W &= -h_1 f(u) + 2h_2(-3 + u(8u - 7u^3 - 2\kappa_0^2 f(u) + 2\varpi_0^2))\,, \\
\tilde{C}_W &= -8h_2\kappa_0^2 f(u) + \frac{4h_2(2+u^2)\varpi_0^2}{f(u)}\,, \\
\tilde{D}_W &= \frac{1}{uf(u)^2}\Big(-\kappa_0^2 f(u)^2(h_1 + h_2(3f(u) + 2\kappa_0^2 u)) \\
&\quad +(h_1 f(u) + h_2(3 + u^4 + 4\kappa_0^2 u f(u)))\varpi_0^2 - 2h_2 u\varpi_0^4\Big)\,, \\
\tilde{E}_W &= -2h_2 u^2 f(u)^2\,,
\end{aligned}
$$

$$
\begin{aligned}
a_1^w &= 2\left(72a_1 - 4b_1 + 8b_2 - 2c_2 + 5c_3 - c_4 + 2c_6 - c_7 + d_3 + d_4\right), \\
a_2^w &= 2\left(-72a_1 + 4b_1 - 12b_2 + 6c_2 + 3c_3 + 9c_4 - 2c_6 + 9c_7 + 3d_1 + 3d_2 - 2d_3 + d_4\right), \\
b_1^w &= \frac{1}{4}\ \left(2[11c_1 + 3(13c_3 + 2c_4 + c_5 - 2(3c_7 + c_8))] + 2(-2d_1 - 5d_2 + 9d_3 + 4d_4)\right. \\
&\quad \left. -36(-6a_2 - e_1) + 8(16b_3 + 22b_4 + f_1) - 5g_1 + 2g_2 + g_3\right)\ , \\
b_2^w &= \frac{1}{4}\left(576a_2 - 32b_2 + 352b_3 + 480b_4 + 64c_1 + 32c_2 + 256c_3 + 80c_4 + 16c_5 - 48c_7 - 32c_8\right. \\
&\quad \left. +8d_1 - 8d_2 + 40d_3 + 32d_4 + 36e_1 + 8f_1 - 5g_1 + 2g_2 + g_3\right), \\
b_3^w &= \frac{1}{4}\left(-576a_1 + 504a_2 + 32b_1 - 128b_2 + 320b_3 + 432b_4 + 62c_1 + 80c_2 + 238c_3 + 132c_4\right. \\
&\quad \left. +14c_5 - 16c_6 + 20c_7 - 28c_8 + 28d_1 + 14d_2 + 18d_3 + 32d_4\right), \\
b_4^w &= -36a_2 - 16b_3 - 24b_4 - c_1 - c_3 + 6c_4 - c_5 + 2(c_7 + c_8) + d_1\ , \\
b_5^w &= 36a_2 + 24b_3 + 32b_4 + 5c_1 + 9c_3 - 2c_4 + c_5 - 10c_7 - 2c_8 - 2d_1 - 3d_2 + 3d_3\ , \\
c_1^w &= \frac{2}{3}\left(-72a_1 - 36a_2 + 4b_1 - 10b_2 - 16b_3 - 20b_4 + c_1 + 4c_2 - 10c_3 + 5c_4 - c_5 - 2c_6 + 3c_7\right. \\
&\quad \left. +2c_8 + d_1 - 2(d_3 + d_4)\right), \\
c_2^w &= -\frac{2}{3}\left(36a_2 + 2b_2 + 16b_3 + 20b_4 - c_1 - 2c_2 + 5c_3 - 4c_4 + c_5 - 2c_7 - 2c_8 - d_1 + d_3 + d_4\right), \\
c_3^w &= -144a_1 + 36a_2 + 8b_1 - 28b_2 + 24b_3 + 32b_4 + 5c_1 + 16c_2 + 23c_3 + 24c_4 + c_5 - 4c_6 + 16c_7 \\
&\quad -2c_8 + 7d_1 + 6d_2 - 2d_3 + 4d_4\ , \\
c_4^w &= -72a_2 - 4b_2 - 48b_3 - 64b_4 - 10c_1 + 4c_2 - 10c_3 + 12c_4 - 2c_5 + 28c_7 + 4c_8 + 7d_1 + 9d_2
\end{aligned}
$$

$$
\begin{aligned}
& -7d_3+2d_4\,, \\
d_1^w &= \frac{1}{4}\left(-108a_2-56b_3-64b_4+c_1-3c_3+22c_4-3c_5+6c_7+6c_8+4d_1-d_2+d_3\right), \\
d_2^w &= \frac{1}{4}\left(-108a_2-56b_3-64b_4+c_1-3c_3+22c_4-3c_5+6c_7+6c_8+4d_1-d_2+d_3-36e_1\right. \\
&\quad \left.-4f_1-3g_1-2g_2-g_3\right), \\
d_3^w &= \frac{1}{4}\left(-108a_2-64b_3-104b_4-11c_1-19c_3+10c_4-7c_5+26c_7+14c_8+8d_1+7d_2-7d_3\right. \\
&\quad \left.-36e_1-8f_1+5g_1-2g_2-g_3\right), \\
d_4^w &= \frac{1}{4}\left(16b_3-16b_4+8c_1+16c_3+8c_4-8c_5-8c_7+16c_8+4d_1-4d_2+4d_3-36e_1-8f_1\right. \\
&\quad \left.+5g_1-2g_2-g_3\right), \\
d_5^w &= \frac{1}{4}\left(-36a_2-16b_3-40b_4-c_1-c_3+6c_4-5c_5+6c_7+10c_8+4d_1+d_2-d_3\right), \\
d_6^w &= -2\left(72a_1-36a_2-4b_1+4b_2-8b_3-16b_4+3c_1+2c_2-4c_3-7c_4-c_5+2c_6+c_7\right. \\
&\quad \left.+2c_8-d_1-d_3-d_4\right), \\
d_7^w &= 2\left(-72a_1+36a_2+4b_1-12b_2+24b_3+32b_4+5c_1+6c_2+12c_3+7c_4+c_5-2c_6-c_7\right. \\
&\quad \left.-2c_8+d_1+d_3+d_4\right), \\
d_8^w &= 4\left(72a_1-36a_2-4b_1+8b_2-16b_3-24b_4-c_1-2c_2+4c_3+5c_4-c_5+2c_6+c_7+2c_8\right. \\
&\quad \left.+d_1+d_3+d_4\right), \\
e_1^w &= 2\left(-72a_1+36a_2+4b_1-12b_2+24b_3+32b_4+5c_1+6c_2+12c_3+7c_4+c_5-2c_6\right. \\
&\quad \left.-c_7-2c_8+d_1+d_3+d_4\right), \\
f_1^w &= \frac{1}{2}\left(-288a_1+180a_2+16b_1-56b_2+120b_3+160b_4+25c_1+32c_2+73c_3+42c_4\right. \\
&\quad \left.+5c_5-8c_6+2c_7-10c_8+8d_1+3d_2+5d_3+8d_4\right), \\
f_2^w &= \frac{1}{2}\left(108a_2-8b_2+72b_3+96b_4+15c_1+8c_2+43c_3+10c_4+3c_5-14c_7-6c_8-3d_2\right. \\
&\quad \left.+7d_3+4d_4\right).
\end{aligned}
$$

Solving EOM

The equation of motion is given by

$$A_x'' + p_1 A_x' + p_0 A_x = \gamma \frac{1}{2f(u)} V(A_x)$$

where

$$\begin{aligned}
&A_W\, A_x'' + C_W A_x' + 2\left(\delta D_1 + D_W\right) A_x \\
&-\partial_u \left(2\delta B_1 A_x' + 2B_W A_x' + C_W A_x + F_W A_x''\right) \\
&+\partial_u^2 \left(A_W A_x + 2E_W A_x'' + F_W A_x'\right) = V(A_x)
\end{aligned}$$

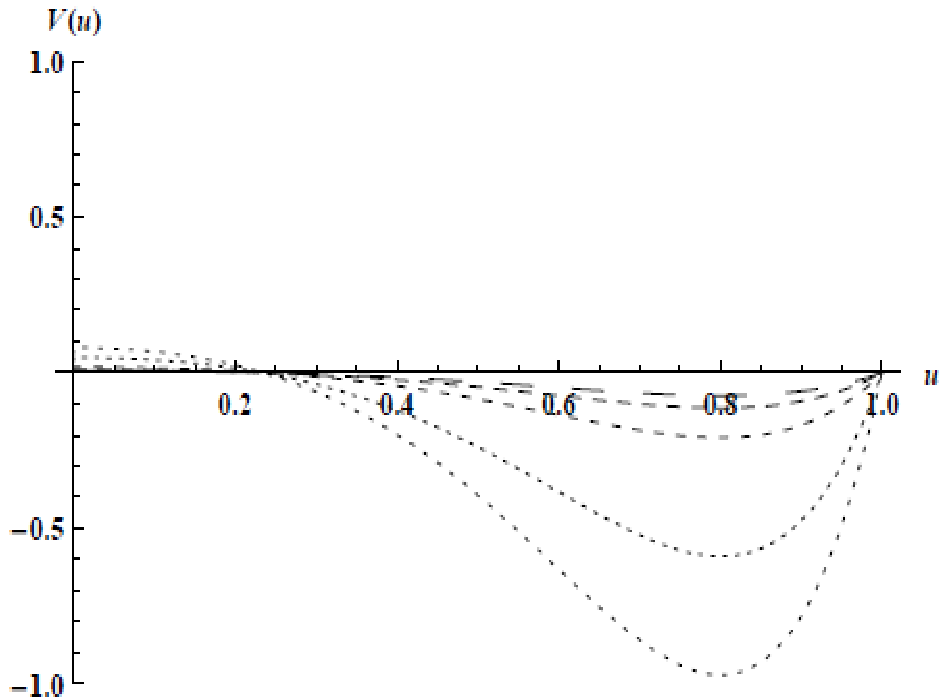
where $B_1 - B_1|_{\gamma\to 0} = \delta B_1$ and $D_1 - D_1|_{\gamma\to 0} = \delta D_1$. First we have the coefficients with no γ-dependence p_0 and p_1, given by

$$p_0 = \frac{\varpi_0^2 - f(u)\kappa_0^2}{u f^2(u)} \quad \text{and} \quad p_1 = \frac{f'(u)}{f(u)}$$

where $\varpi_0 = \omega/(2\pi T)$ and $\kappa_0 = q/(2\pi T)$. For the coefficients originating from the F^2 term in the action of the gauge field, we obtain

$$\begin{aligned}
B_1 &= \frac{K(u) f(u) L^7(u)}{P(u)}\,, \\
D_1 &= -K(u) P(u) L^7(u) \left[\frac{\varpi^2 - f(u) K^2(u) \kappa^2}{u f(u) K^2(u)}\right]
\end{aligned}$$

where $\varpi = \omega R^2/(2r_0)$ and $\kappa = qR^2/(2r_0)$.


V(u)
1.0
0.5
−0.5
−1.0
0.2
0.4
0.6
0.8
1.0
u

Singularity structure of the equation at the horizon

Set $x = 1 - u$, so that the singularity is at $x = 0$, then insert the functional form $A_x = x^\beta$. We obtain the indicial equation:

$$\beta^2 + \left(\frac{\omega}{4\pi T}\right)^2 = 0$$

The hydrodynamic regime

$$A_x(u) = A_0(u) + \gamma A_1(u) \; = \; [1-u]^{-\sigma} \left(\phi_0(u) + \gamma \phi_1(u) \right)$$

where $\sigma = i\omega/(4\pi T)$. We now write

$$\phi_{0,1}(u) = h_{0,1}(u) + \sigma g_{0,1}(u).$$

The full solution to linear order in γ and σ:

$$A_x(u) = [1-u]^{-\sigma} \left(\overline{C} + \sigma \left\{ \overline{D} + \overline{C} \left(1 + \gamma \left[\frac{185}{4} + 2\tilde{\alpha} \right] \right) u \right\} \right)$$

where

$$\tilde{\alpha} = 216a_2 + 144b_3 + 192b_4 + 30c_1 + 54c_3 - 12c_4 + 6c_5 - 60c_7 - 12c_8 - 12d_1 - 18d_2 + 18d_3 - 36e_1 - 8f_1 + 5g_1 - 2g_2 - g_3 + h_2$$

If we call the boundary value of the field A_T, the on-shell action is given by

$$S_{\text{total}} = -\frac{\tilde{N}^2 r_0^2}{16\pi^2 R^4} \int \frac{d^4k}{(2\pi)^4} \int_0^1 du \left[\frac{1}{2} A_{-k} \mathcal{L}\, A_k + \partial_u \Psi\right]$$

where $\mathcal{L} A_k = 0$ is the EOM, and Ψ is a boundary term. On the on-shell action, the only surviving term is the boundary term, as we expect from holography. This is given by

$$\Psi = (B_1 + \gamma B_W - \gamma A_W) A_k' A_{-k} + \tfrac{\gamma}{2}(C_W - A_W') A_k A_{-k} - \gamma E_W' A_k'' A_{-k} + \gamma E_W A_k'' A_{-k}' - \gamma E_W A_k''' A_{-k} + \gamma E_W \left(p|_1 A_k' + 2p_0 A_k\right) A_{-k}' - \gamma \tfrac{F_W'}{2} A_k' A_{-k}$$

We only get contributions from B_1, B_W and F_W inside Ψ. Remembering that $r_0 = \pi T R^2(1 - 265/16\gamma)$, we obtain that the conductivity is then corrected by a factor

$$1 + \gamma\,(\alpha - 10)$$

where

$$\alpha = \tilde{\alpha} - h_1 - 3h_2$$

Electric charge conductivity for AdS-Schw. B.H. times the sphere

$$\sigma = \sigma_\infty \left(1 + \frac{\zeta[3]}{8} C \, \lambda^{-3/2}\right) \; ,$$

where $\sigma_\infty = e^2 \, N^2 \, T/(16\pi)$ and

$$C = C_{C^4} - 10 + C_{C^2(\nabla F)^2} + C_{C^2 \mathcal{T}^2} + C_{C^3 \mathcal{T}^1} \, .$$

The contribution of each set of operators are:

- $C_{C^4} = 44/3$
- $C_{C^2(\nabla F)^2} = 12797/9$
- $C_{C^2 \mathcal{T}^2} = 2490/9$
- $C_{C^3 \mathcal{T}^1} = -336/9$

Gathering all these it gives $C = 14993/9 \approx 1665.89$. Thus

$$\sigma = \frac{e^2 \, N^2 \, T}{16\pi} \left(1 + \frac{\zeta(3)}{8} \, \frac{14993}{9} \, \lambda^{-3/2}\right) \; ,$$

Examples:

for $\lambda = 100$ the enhancement $\frac{\zeta[3]}{8} \frac{14993}{9} \lambda^{-3/2}$ is about 0.25

for $\lambda = 1000$ the enhancement $\frac{\zeta[3]}{8} \frac{14993}{9} \lambda^{-3/2}$ is about 0.0079

$$S^{(3)}_{\mathcal{R}^4} = \frac{\alpha'^3 g_s^{3/2}}{32\pi G} \int d^{10}x \int d^{16}\theta \sqrt{-g} f^{(0,0)}(\tau,\bar{\tau}) \times$$
$$[(\theta\Gamma^{mnp}\theta)(\theta\Gamma^{qrs}\theta)\mathcal{R}_{mnpqrs}]^4 + c.c.$$

$$\mathcal{T}_{abcdef} = i\nabla_a F^+_{bcdef}$$
$$+\frac{1}{16}\left(F^+_{abcmn}F^+_{def}{}^{mn} - 3F^+_{abfmn}F^+_{dec}{}^{mn}\right)$$

Type IIB supergravity describes the limits $\lambda \to \infty$ and $N_c \to \infty$, with $1 \ll \lambda \ll N_c$. The high derivative corrections we consider above give finite λ corrections but still $N_c \to \infty$. The modular form $f^{0,0}(\tau,\bar{\tau})$ of Eq.(4) holds for all values of the string coupling, and is given by

$$f^{(0,0)}(\tau,\bar{\tau}) = 2\zeta(3)\tau_2^{3/2} + \frac{2\pi^2}{3}\tau_2^{-1/2} + 8\pi\tau_2^{1/2} \times$$
$$\sum_{m\neq 0, n\geq 0} \frac{|m|}{|n|} e^{2\pi i m n \tau_1} K_1(2\pi|mn|\tau_2)\,, \qquad (14)$$

where K_1 is the modified Bessel function of second kind which comes from non-perturbative D-instantons contributions. Since in the present setup $\tau_1 = 0$ and $\tau_2 = g_s^{-1}$,

$f^{(0,0)}(\tau, \bar{\tau})$ becomes

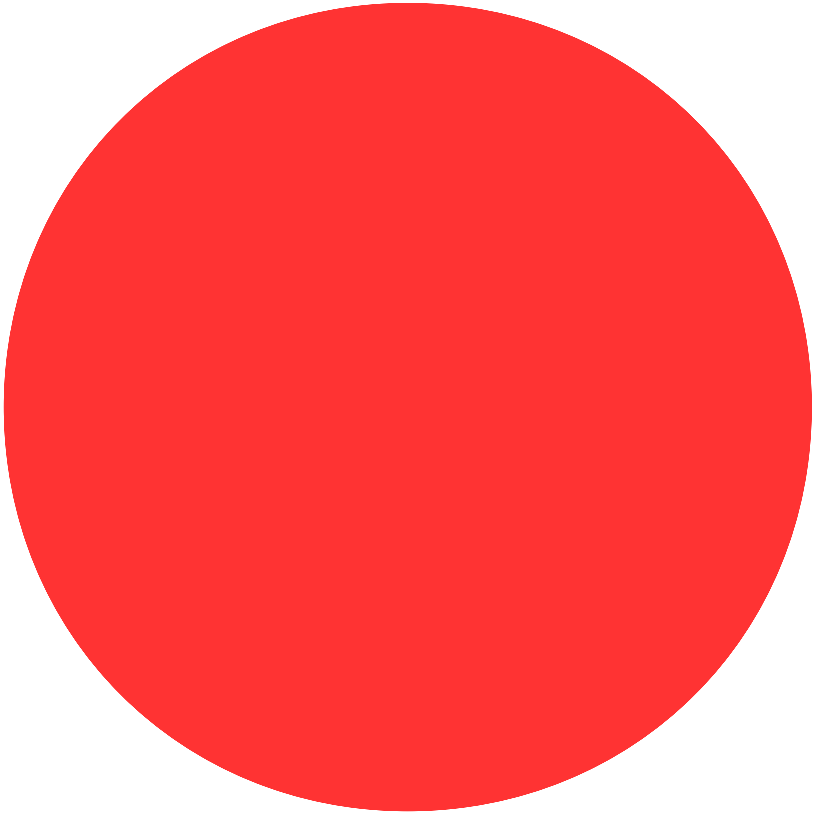
$$2(4\pi N_c)^{3/2}\left(\frac{\zeta(3)}{\lambda^{3/2}}+\frac{\lambda^{1/2}}{48N_c^2}+\frac{e^{-8\pi^2 N_c/\lambda}}{4\pi^{1/2}N_c^{3/2}}\right), \quad (15)$$

where for small g_s, non-perturbative contributions reduce to the third term. Recall that the dilaton gets only $\mathcal{O}(\alpha'^3)$ corrections, thus the electrical conductivity is modified as

$$\sigma = \sigma_0 + \frac{C\sigma_0}{8}\left(\frac{\zeta(3)}{\lambda^{3/2}}+\frac{\lambda^{1/2}}{48N_c^2}+\frac{e^{-8\pi^2 N_c/\lambda}}{4\pi^{1/2}N_c^{3/2}}\right), \quad (16)$$

where $\sigma_0 = \frac{e^2 T N_c^2}{16\pi}$. At this point one may wonder whether this result is comparable with those obtained from lattice QCD. Obviously, any statement in the context of the present work has to be considered with several caveats, coming from differences between QCD and $\mathcal{N} = 4$ SYM. Said that, it is possible to make contact with lattice QCD at some extent. We must take into account that in lattice calculations $N_c = 3$ and there are other differences with respect to the large N_c of $\mathcal{N} = 4$ SYM plasma. In a recent estimation for the electrical conductivity [12] it was found $\sigma \simeq 0.4\, e^2 T$, above T_c of quenched lattice QCD. A more recent calculation [13] shows that $1/3\, e^2 T \leq \sigma \leq e^2 T$ from the vector current correlation function for light valence quarks in the deconfined phase of quenched lattice QCD at $T = 1.45 T_c$. It is worth noting that from lattice computations at temperatures about 1.5 to 2 T_c, the values of $\alpha_s = g^2_{YM}/4\pi$ are between 0.3 and 0.4, where these values were obtained by matching the Debye mass screening in QCD and in $\mathcal{N} = 4$ SYM theory at finite T. Let us use the parametrization $\sigma = \rho\, e^2 T$ and extract ρ from our Eq.(16). We naively set $N_c = 3$, and evaluate the electrical conductivity for different values of $\lambda = 11.3, 15.08, 6\pi$ which lead to $\alpha_s = 0.3, 0.4, 0.5$, thus Eq.(16) gives 1.64, 1.28 and 1.101, respectively, for ρ in the electrical conductivity. On the other hand, using these values of λ and N_c it is only possible to discuss the zero frequency limit of the photoemission rates, since they are well beyond the range of validity of the approximations considered in this work. Even though, one may try to see what happens for more suitable values like $\lambda = 50$ and $N_c = 100$ or any of the λ values in figure 1, varying N_c from, say 100.000 to 100. In those cases, the large N_c and finite N_c curves coincide for each λ, therefore, being finite N_c effects negligible in the range where our approximations hold.

AdS/CFT: Thermalisation



Thanks!